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A constructive existence proof for first level formal solutions of meromorphic differential equations

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0. Introduction

In [1], [2], [3] the author has introduced and studied a generalized type of formal solutions (*formal fundamental solutions of first level*; see Section 3 for the definition) for meromorphic differential equations. Compared to the usual kind of formal solutions, first level formal solutions have the advantage of always being "summable" in terms of Laplace integrals or, equivalently, generalized factorial series and in this way generating a family of (proper) solutions of the differential equation with natural asymptotic properties in certain sectors [3]. On the other hand, the existence proof for first level formal solutions, given in [1], [2], made use of the Asymptotic Existence Theorem as well as a theorem on the existence of differential equations with a prescribed Stokes' phenomenon. In the present paper we give a completely different proof for the existence of first level formal fundamental solutions which is much more elementary than the original one, since it only uses some results from the *formal* theory of meromorphic differential equations and Banach's Fixed Point Theorem. At the same time the proof is completely constructive and may therefore be made a basis for actually calculating such formal solutions, although it is very likely that there may be more effective ways for calculating them than following all the steps of the proof.

The main idea in the proof is to obtain an improved version of a well-known result upon formal block-diagonalization of a meromorphic differential equation: Let the leading term of the coefficient matrix (in the Laurent expansion about a pole) be a direct sum of blocks such that each two of them have no common eigenvalue. Then a formal meromorphic transformation may be constructed which block-diagonalizes the given equation, however the resulting equation is, in general, a *formal* equation in the sense that the resulting Laurent series for its coefficient matrix does not converge. It is shown in this paper that a modification of the usual construction of the formal transformation leads to a converging (in fact terminating) Laurent series for the resulting differential equation, and at the same time an estimate upon the coefficients of the formal transformation is achieved.