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Stably extendible vector bundles over the real projective spaces and the lens spaces

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ABSTRACT. Let RP^n be the n-dimensional real projective space and let $L^n(q)$ be the $(2n+1)$-dimensional standard lens space mod q. The purpose of this paper is to prove that a complex k-dimensional vector bundle ζ over RP^n is stably equivalent to a sum of k complex line bundles if ζ is stably extendible to RP^m for every $m > n$, to prove that a real k-dimensional vector bundle ζ over $L^n(3)$ is stably equivalent to a sum of $[k/2]$ real 2-plane bundles if ζ is stably extendible to $L^m(3)$ for every $m > n$ and to study non stable extendibility of complex vector bundles over $L^n(4)$.

1. Introduction

Let F denote the real field R, the complex field C or the quaternion field H. Let X be a CW-complex and A be a subcomplex. A k-dimensional F-vector bundle ζ over A is called extendible (respectively stably extendible) to X, if there exists a k-dimensional F-vector bundle α over X whose restriction to A is equivalent (respectively stably equivalent) to ζ as F-vector bundles, that is, if the restriction $\alpha|A$ of α to A is isomorphic to ζ (respectively $(\alpha|A) \oplus \varepsilon^n$ is isomorphic to $\zeta \oplus \varepsilon^n$ for some trivial F-vector bundle ε^n over A), where $\oplus$ denotes the Whitney sum (cf. Schwarzenberger [14] and Imaoka-Kuwana [4]).

In the following we say simply a vector bundle instead of an R-vector bundle.

Concerning stably extendible F-vector bundles for $F = C$ and R, the following results are known.

THEOREM (Schwarzenberger (cf. [3], [14], [2], [13])). *Let $F = C$ or R. If a k-dimensional F-vector bundle ζ over FP^n is stably extendible to FP^m for each $m > n$, then ζ is stably equivalent to a sum of k F-line bundles.*

In the original results of Schwarzenberger, the F-vector bundles are assumed to be extendible, but the results are also valid for the stably extendible F-vector bundles.

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