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MEAN CURVATURES FOR ANTIHOLOMORPHIC p-PLANES IN SOME ALMOST HERMITIAN MANIFOLDS

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1. Let (M, g) be an n-dimensional Riemannian manifold with (positive definite) metric tensor g. We denote by $K(x, y)$ the sectional curvature for a 2-plane spanned by x and y. Let m be a point of M and π a q-plane at m. An orthonormal basis $\{e_i;\ i=1, 2, \cdots. n\}$ such that $e_1, e_2, \cdots, e_q$ span π is called an adapted basis for π. Then

$$\rho(\pi)=\frac{1}{q(n-q)}\sum_{a=q+1}^{n}\sum_{\alpha=1}^{q}K(e_\alpha, e_a) \tag{1}$$

is independent of the choice of an adapted basis for π and is called by S. Tachibana [5] the *mean curvature* $\rho(\pi)$ for π.

Before formulating the main theorem of this paper, we give some propositions for the mean curvature.

PROPOSITION A (S. Tachibana [5]). *In an $n(>2)$-dimensional Riemannian manifold $(M. g)$, if the mean curvature for a q-plane is independent of the choice of q-planes at each point, then*

(i) *for $q=1$ or $n-1$, (M, g) is an Einstein space;*
(ii) *for $1<q<n-1$ and $2q\neq n$, (M, g) is of constant curvature;*
(iii) *for $2q=n$, (M, g) is conformally flat.*

The converse is true.

Taking holomorphic $2p$-planes instead of q-planes, an analogous result in Kähler manifolds is obtained:

PROPOSITION B (S. Tachibana [6] and S. Tanno [7]). *In a Kähler manifold (M, g, J), $n=2k\geqq 4$, if the mean curvature for a holomorphic $2p$-plane is independent of the choice of holomorphic $2p$-planes at a point m, then*

(i) *for $1\leqq p\leqq k-1$ and $2p\neq k$ (M, g, J) is of constant holomorphic sectional curvature at m;*
(ii) *for $2p=k$, the Bochner curvature tensor vanishes at m.*

The converse is true.

Remark that the case $n=2$ is trivial and that Proposition B can be formulated *globally.* In this case, the converse of (ii) is true if and only if the scalar cur-

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