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KŌDAI MATH. SEM. REP.
27 (1976), 313–319

SEMI-INVARIANT IMMERSIONS

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§ 1. Introduction.

Let $\tilde{M}$ be a differentiable manifold and let $\tilde{F}$ be a tensor field of type (1, 1) defined on $\tilde{M}$. If M is a submanifold immersed in $\tilde{M}$, M is said to be an invariant submanifold if the tangent space to M at each point of M is invariant under the endomorphism $\tilde{F}$. If $\tilde{M}$ is a complex manifold and $\tilde{F}$ is the almost complex structure on $\tilde{M}$ then the invariant submanifolds of $\tilde{M}$ are just the complex submanifolds. (See, Schouten and Yano [4]). If $\tilde{M}$ is a normal contact (or Sasakian) manifold with $(\tilde{F}, \tilde{\xi}, \tilde{\eta})$ as the almost contact structure on $\tilde{M}$, then there does not exist an invariant submanifold M with $\tilde{\xi}$ normal to M. (See § 4). Invariant submanifolds have been studied by many people. (See, Kubo [2], Schouten and Yano [4], Yano and Okumura [9], [10]).

The purpose of this paper is to study submanifolds M of $\tilde{M}$ for which there is a distribution D that is nowhere tangent to M and such that the subspace spanned by D and the tangent space to M is invariant under $\tilde{F}$ and $\tilde{F}D$ is tangent to M at each point of M. (See, Tashiro [6]).

§ 2. Preliminaries.

Let M be a differentiable manifold. A tensor field F of type (1, 1) on M defines an *almost complex* structure if

$$F^2 = -I,$$

in which case M is of even dimension. This almost complex structure is *integrable*, i. e., M is complex, if $[F, F]=0$, where $[F, F]$ is the Nijenhuis tensor of F defined by

$$[F, F](X, Y) = [FX, FY] - F[FX, Y] - F[X, FY] + F^2[X, Y].$$

Here X and Y are vector fields on M. A Riemannian metric g is a Hermitian metric for F if

$$g(FX, FY) = g(X, Y).$$

Received June 24, 1974.
* Work done under partial support by NSF Grant No. 36684.