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GLOBAL SOLUTIONS OF CERTAIN FOURTH ORDER DIFFERENTIAL EQUATIONS

Dedicated to Professor Yûsaku Komatu on his 60th birthday

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§ 1. Introduction.

In this paper, we consider homogeneous and nonhomogeneous fourth order ordinary differential equations of the form

$$\frac{d^4u}{dz^4}-\left(z\frac{d^2u}{dz^2}+\lambda\frac{du}{dz}\right)=0, \tag{1.1}$$

$$\frac{d^4v}{dz^4}-\left(z\frac{d^2v}{dz^2}+\lambda\frac{dv}{dz}\right)=b(z), \tag{1.2}$$

where u and v are unknown functions of z, z is the complex independent variable, λ is a constant and $b(z)$ is a known function of z. These equations are related to the Orr-Sommerfeld and its adjoint equations, we call them together Orr-Sommerfeld type equations that play the fundamental role in the theory of hydrodynamic stability of viscous fluids. The Orr-Sommerfeld type equations are of the form

$$\varepsilon^2\frac{d^4\varphi}{dx^4}-\left\{p_3(x,\varepsilon)\frac{d^2\varphi}{dx^2}+p_2(x,\varepsilon)\frac{d\varphi}{dx}+p_1(x,\varepsilon)\varphi\right\}=0, \tag{1.3}$$

where ε is a small positive parameter and $p_i(x,\varepsilon)$ $(i=1,2,3)$ can be expanded asymptotically in power series of ε with holomorphic coefficients. Except for a small neighborhood of turning point x where $p_3(x,0)=0$, asymptotic solutions of (1.3) were obtained by the W-K-B type approximation, Nishimoto [1]. On the other hand, asymptotic expansions in the direct neighborhood of a turning point are constructed by either the related equation method or the matching procedure. If we apply the matching method to the equation (1.3) in the neighborhood of its simple turning point which is assumed to be at the origin, according to Nishimoto [2], page 238-239 with $n=4$, $m=2$ and $q=1$, it becomes necessary to study the equations

$$\frac{d^4u_0}{dx^4}-\left(ax\frac{d^2u_0}{dx^2}+b\frac{du_0}{dx}\right)=0,$$

$$\frac{d^4u_i}{dx^4}-\left(ax\frac{d^2u_i}{dx^2}+b\frac{du_i}{dx}\right)=\sum_{k=1}^{i}b_{1k}u_{i-k}+b_{2k}\frac{du_{i-k}}{dx}+b_{3k}\frac{d^2u_{i-k}}{dx^2}+b_{4k}\frac{d^3u_{i-k}}{dx^3}.$$

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