G. D. DIKSHIT
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ON ABSOLUTE RIESZ SUMMABILITY FACTORS OF FOURIFR SERIES

BY G. D. DIKSHIT

1. Definitions and Notations.

Let $\sum u_n$ be an infinite series and let $0=\lambda_0<\lambda_1<\cdots<\lambda_n\to\infty$. For $k\geqq 0$, we write

$$R^k(\lambda_m)=\sum_{n=1}^{m-1}\left(1-\frac{\lambda_n}{\lambda_m}\right)^k u_n .$$

The series $\sum u_n$ is said to be absolutely summable by Riesz discrete method (R^*, λ_n, k), or summable $|R^*, \lambda_n, k|$, if $\{R^k(\lambda_n)\}$ is of bounded variation. When the 'order' of summation $k=1$, the method of summation is equivalent to the usually known Riesz method $|R, \lambda_n, 1|$. In this specialcase the method is also sometimes known as the method $|R, \mu_n|$, or the method $|\bar{N}, \mu_n|$, where $\{\mu_n\}=\{\lambda_n-\lambda_{n-1}\}$. In this paper we are concerned with this special case and we will denote the method of summation by $|R, \lambda_n, 1|$. Thus the series $\sum u_n$ is summable $|R, \lambda_n, 1|$ if

$$\sum_1^\infty |\Delta R^1(\lambda_m)|=\sum_{m=1}^\infty\left(\frac{1}{\lambda_m}-\frac{1}{\lambda_{m+1}}\right)\left|\sum_{n=0}^m \lambda_n u_n\right|<\infty .$$

When $\{\lambda_n\}=\{n\}$, the method is the same as the Cesàro method $|C, 1|$, and when $\{\lambda_n\}=\{\log n\}$ the method is called the logarithmic method.

Let $f(t)$ be a Lebesgue integrable 2π-periodic function and let the Fourier series of $f(t)$ be given by

$$f(t)\sim\frac{a_0}{2}+\sum_1^\infty(a_n\cos nt+b_n\sin nt)=\sum_0^\infty A_n(t) .$$

We set

$$\phi(t)=\frac{1}{2}\{f(x+t)+f(x-t)-2f(x)\} ,$$

and throughout the paper we write to denote

$$\Phi(t)=\int_0^t|\phi(u)|\,du ,$$

$$p_n=\int_{1/n}^\pi\frac{|\phi(u)|\,du}{u} ,$$

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