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MORSE FUNCTIONS ON SOME ALGEBRAIC VARIETIES

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1. Introduction.

For any n-tuple of integers $a=(a_1, \cdots, a_n)$ $(a_i \geqq 2)$ Brieskorn variety $\sum(a)$ is, by definition, a real algebraic variety given by the following equations in $z=(z_1, \cdots, z_n) \in \mathbf{C}^n$:

$$z_1^{a_1}+\cdots+z_n^{a_n}=0\,,$$

$$z_1\bar{z}_1+\cdots+z_n\bar{z}_n=1\,,$$

$\sum(a)$ is known to be the boundary of some parallelizable $(2n-2)$-manifold while, if $n \geqq 4$, any homotopy $(2n-3)$-sphere being the boundary of the parallelizable manifold becomes diffeomorphic to some Brieskorn variety. Moreover, in case where $\sum(a)$ is the homotopy sphere, E. Brieskorn [1] and H. Hirzebruch and K. H. Mayer [3] have shown that the diffeomorphism type of $\sum(a)$ can be completely classified in terms of $a=(a_1, \cdots)$ using the famous theory due to M. Kervaire and J. Milnor [2]. In the present paper we shall show that two Brieskorn varieties

$$\sum(a_2, a_3, \cdots, a_n) \quad \text{and} \quad \sum(a_1, a_3, \cdots, a_n) \qquad (n \geqq 3)$$

are cobordant and this cobordism is realized by a real algebraic variety W defined by the following equations in $(z, t) \in \mathbf{C}^n \times [0, 1]$:

$$f(z)=z_1^{a_1}+\cdots+z_n^{a_n}=0\,, \tag{1}$$

$$g(z, t)=tz_1+(1-t)z_2=0\,, \tag{2}$$

$$h(z)=|z|^2-1=z_1\bar{z}_1+\cdots+z_n\bar{z}_n-1=0\,. \tag{3}$$

Besides, in many cases the real valued function t on W becomes a Morse function, hence the study of the function t gives us the information on the homotopy type of W. More precisely, we shall prove the following theorem.

THEOREM. *In case $n \geqq 3$ and $a_2 > a_1$, W is a smooth $(2n-4)$-manifold which gives a cobordism between $\sum_1=\sum(a_2, a_3, \cdots, a_n)$ and $\sum_2=\sum(a_1, a_3, \cdots, a_n)$. If $10 \geqq a_2 > a_1 = 2$ or $a_2 > a_1 > 2$, then t is a Morse function on W. The Morse index at the critical point (z, t) is given by*

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