D. S. GOEL
KODAI MATH. SEM. REP.
26 (1975), 187—193

ALMOST TANGENT STRUCTURES

BY D. S. GOEL

0. Let M be a differentiable manifold of class C^∞ and of dimension $2n$. A (1, 1) tensor field J of rank n on M such that $J^2=0$ defines a class of conjugate G-structures on M. A group G for a representative structure consists of all matrices of the form

$$\begin{bmatrix} A & 0 \\ B & A \end{bmatrix} \tag{0.1}$$

where A, B are matrices of order $n\times n$ and A is non-singular. This structure is called an almost tangent structure [4]. Suppose that such a structure is defined on M then M is called an almost tangent manifold. A (1, 1) tensor field J on M can be defined by specifying its components to be

$$J_0=\begin{bmatrix} 0 & 0 \\ I_n & 0 \end{bmatrix} \tag{0.2}$$

relative to any adapted frame. If $\sigma=X_1, \cdots, X_{2n}$ is any adapted moving frame defined at a given point $m\in M$, then $JX_a=X_{a+n}$, $JX_{a+n}=0$ $(a=1, \cdots, n)$. The tensor field J has constant rank n and it satisfies the equation $J^2=0$. Conversely any such tensor field J determines an almost tangent structure on M [5]. The (1, 1) tensor field J on an almost tangent manifold M determines a linear mapping $J_m: v\to(Jm)v$ on each tangent vector space T_mM. The function $\operatorname{Ker} J: m\to$ kernel J_m is an n-dimensional distribution on M. If σ is an adapted moving frame at any given point $m\in M$, then the vector fields $X_{n+1}, \cdots, X_{2n}$ form a local basis for the distribution $\operatorname{Ker} J$ at m.

In this paper we shall study the conditions under which an almost tangent structure is integrable, and show that the group of automorphisms of such a structure is not necessarily a Lie group even on a compact manifold.

1. Suppose that we have any G-structure on a manifold M of dimension n with adapted fram bundle $P(M, G)$. Let θ be the canonical 1-form on $P(M, G)$ with values in R^n and ω the connection form of a given linear connection on P. If $\Theta=D\theta$ is the torsion form then the torsion tensor $T(\Theta)$ has values in $V=$ $=R^n\otimes\overset{2}{\wedge}R_n$ and is of type $R=\mu\otimes\overset{2}{\wedge}\mu^*$ where μ is a representation of G in R^n defined by the matrix multiplication. We denote $W=L(G)\otimes R_n$, where $L(G)$ is

Received June 15, 1973.