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KŌDAI MATH. SEM. REP.
26 (1975), 160—175

ON $(f, g, u_{(k)}, \alpha_{(k)})$-STRUCTURES

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§0. Introduction.

Yano and Okumura [6] have studied hypersurfaces of a manifold with (f, g, u, v, λ)-structure. These submanifolds admit under certain conditions what we call an $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In particular, a hypersurface of an even-dimensional sphere carries an $(f, g, u_{(k)}, \alpha_{(k)})$-structure (see also Blair, Ludden and Okumura [2]). Submanifolds of codimension 2 in an almost contact metric manifold also admit the same kind of structure (see Yano and Ishihara [5]).

The main purpose of the present paper is to study the $(f, g, u_{(k)}, \alpha_{(k)})$-structure and hypersurfaces of an even-dimensional sphere. In §1, we define and discuss $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure and $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In §2, we recall the definition of (f, g, u, v, λ)-structure and give examples of the manifold with $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In §3, we study non-invariant hypersurfaces of a manifold with normal (f, g, u, v, λ)-structure. In the last section, we study hypersurfaces of an even-dimensional sphere S^{2n} under certain conditions by using of the following theorem proved by Ishihara and one of the present authors [3]:

THEOREM A. *Let (M, g) be a complete and connected hypersurface immersed in a sphere $S^{m+1}(1)$ with induced metric g_{ji} and assume that there is in (M, g) an almost product structure $P_i{}^h$ of rank p such that $\nabla_j P_i{}^h=0$. If the second fundamental tensor H_{ji} of the hypersurface (M, g) has the form $H_{ij}=aP_{ji}+bQ_{ji}$, a and b being non-zero constants, where $P_{ji}=P_j{}^t g_{it}$ and $Q_{ji}=g_{ji}-P_{ji}$, and, if $m-1\geqq p\geqq 1$, then the hypersurface (M, g) is congruent to the hypersurface $S^p(r_1)\times S^{m-p}(r_2)$ naturally embedded in $S^{m+1}(1)$, where $1/r_1{}^2=1+a^2$ and $1/r_2{}^2=1+b^2$.*

§1. $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure.

Let M be an m-dimensional differentiable manifold of class C^∞. We assume there exist on M a tensor field f type (1,1), vector fields U, V and W, 1-forms u, v and w, functions α, β and γ satisfying the conditions (1.1)～(1.7):

$$f^2X=-X+u(X)U+v(X)V+w(X)W \tag{1.1}$$

for any vector field X,

Received June 4, 1973.