M. SAKAI
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CONTINUOUS LINEAR FUNCTIONALS ON THE SPACE OF BOUNDED HARMONIC FUNCTIONS

BY MAKOTO SAKAI

Introduction.

Let $X=(X, \| \|)$ be a Banach space, and let T be a continuous linear functional on X. The norm of T is defined by $\|T\|=\sup_{x\in X^1}|T(x)|$, where X^1 is the set of points $x\in X$ such that $\|x\|\leqq 1$. If $y\in X^1$ satisfies $T(y)=\|T\|$, y is called an extremal point (an extremal function if X is a funcion space) of T. The following fundamental assertions are known:

(i) If X is reflexive, then for every continuous linear functional T there exist extremal points of T.

(ii) If X is strictly convex, then for every continuous linear functional T $(\neq 0)$ there exists at most one extremal point of T.

Let $HB(R)$ be the Banach space of all bounded harmonic functions u on a Riemann surface R with the supremum norm:

$$\|u\|_R=\sup_{z\in R}|u(z)|.$$

Then $HB(R)$ is neither reflexive nor strictly convex, in general. Hence there needs the special discussion to obtain the existence and uniqueness theorem of the extremal problems of continuous linear functionals on $HB(R)$.

In this paper we shall deal with the extremal problems of continuous linear functionals on $HB(R)$ and their applications to analytic mappings. In § 1 we give the existence and uniqueness theorem of extremal functions of continuous linear functionals of $HB(R)$. To do this, we use the Wiener compactification of Riemann surfaces and the Riesz representation theorem. The definition of absolutely continuous linear functionals is given in § 2. Linear functionals which appear in function theory are usually absolutely continuous. In § 3 we are concerned with the extensions of continuous linear functionals. As a corollary we see that if $HB(R)$ is of infinite dimension, then $HB(R)$ is not separable. § 4 deals with the so-called harmonic lengths. We give two examples of cycles whose extremal functions of harmonic lengths are not determined uniquely. In the last section, § 5, we discuss applications of the extremal problems to analytic mappings.

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