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QUASI-NORMAL ANALYTIC SPACES, II

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In [2], we have discussed relations between three kinds of analytic sheaves on an analytic space, that is, $\mathcal{O}$, $\mathcal{O}'$ and $\tilde{\mathcal{O}}$ which are, respectively, the sheaf of germs of holomorphic functions, the sheaf of germs of continuous and weakly holomorphic functions, and the sheaf of germs of weakly holomorphic functions.

The present paper is a continuation of [2]. § 1 is devoted to a result concerning the product of analytic spaces, which will be used for dealing with an example in § 3. In § 2, we discuss a quasi-normality condition in the case of subvarieties which consist of certain submanifolds; § 3 contains some examples.

The notations and terminology of [2] will be used without any specific mention.

§ 1. Quasi-normality of product spaces.

We prove the following

THEOREM 1. *Let $X=X_1\times\cdots\times X_m$ be the Cartesian product of analytic spaces X_i, $i=1,\cdots,m$. Let $p=(p_1,\cdots,p_m)\in X$. Then, X is quasi-normal at p if and only if X_i are quasi-normal at p_i for all i.*

Proof. It is sufficient to treat the case in which $m=2$. Let $V=V_1\times V_2$ where V_i are neighborhoods of p_i which are subvarieties of open subsets D_i of $\boldsymbol{C}^{n_i}$, p_i being origins of $\boldsymbol{C}^{n_i}$, $i=1,2$. Assume that V is quasi-normal at $p=(p_1,p_2)$, and let $\boldsymbol{f}\in{}_{V_1}\mathcal{O}'_{p_1}$. Choose a representative f of $\boldsymbol{f}$ which is continuous and weakly holomorphic on $V_1\cap\Delta_1$ where Δ_1 is a suitable neighborhood of p_1 in D_1. Let $\Delta=\Delta_1\times\Delta_2$ where Δ_2 is a neighborhood of p_2 in D_2, and let π denote the projection of $V\cap\Delta$ onto $V_1\cap\Delta_1$. Since

$$\mathcal{R}(V\cap\Delta)=(\mathcal{R}(V_1)\cap\Delta_1)\times(\mathcal{R}(V_2)\cap\Delta_2),$$

we see that $f\circ\pi$ is continuous and weakly holomorphic on $V\cap\Delta$, hence holomorphic on $V\cap\Delta'$, where $\Delta'=\Delta'_1\times\Delta'_2$ with $\Delta'_i\subset\Delta_i$. From this follows that f is holomorphic on $V_1\cap\Delta'_1$, which implies that $\boldsymbol{f}\in{}_{V_1}\mathcal{O}_{p_1}$.

Conversely, let V_i be quasi-normal at p_i, $i=1,2$. We can choose neighborhoods Δ_i of p_i in D_i so that $V_i\cap\Delta_i$ are quasi-normal spaces; this is possible, because the set of points where a space is quasi-normal is open ([2], p. 182). Let $\boldsymbol{F}\in{}_V\mathcal{O}'_p$. There exist neighborhoods Δ'_i of p_i, $\Delta'_i\subset\Delta_i$, and a representative F of $\boldsymbol{F}$ such that F is

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