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ON INTRINSIC STRUCTURES SIMILAR TO THOSE INDUCED ON S^{2n}

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1. In [1] Yano and the authors studied submanifolds of codimension 2 of almost complex manifolds and hypersurfaces of almost contact manifolds. In both cases the structure on the ambient space induced the same structure on the submanifold. The induced structure consists of a tensor field f of type (1, 1), vector fields E, A, 1-forms η, α and a function λ satisfying

$$\begin{aligned} & f^2=-I+\eta\otimes E+\alpha\otimes A, \\ & \eta\circ f=\lambda\alpha, \qquad \alpha\circ f=-\lambda\eta, \\ & fE=-\lambda A, \qquad fA=\lambda E, \\ & \eta(E)=1-\lambda^2, \qquad \alpha(E)=0, \\ & \eta(A)=0, \qquad \alpha(A)=1-\lambda^2. \end{aligned} \tag{1}$$

Moreover the metric g induced from a metric compatible with the structure on the ambient space satisfies

$$\begin{aligned} & g(X, E)=\eta(X), \qquad g(X, A)=\alpha(X), \\ & g(fX, fY)=g(X, Y)-\eta(X)\eta(Y)-\alpha(X)\alpha(Y). \end{aligned} \tag{2}$$

It is well known that on an almost complex manifold or an almost contact manifold there exists a metric compatible with the given structure, i.e. we have an almost Hermitian structure or an almost contact metric structure. However given a $2n$-dimensional manifold M^{2n} with tensors $(f, E, A, \eta, \alpha, \lambda)$ satisfying equations (1), we show in section 2 that there does not in general exist a Riemannian metric on M^{2n} satisfying equations (2). Thus to study manifolds with an intrinsically defined $(f, E, A, \eta, \alpha, \lambda)$-structure from the standpoint of Riemannian geometry it is necessary to assume the existence of a Riemannian metric satisfying equations (2).

The even-dimensional spheres are clearly examples of manifolds with an $(f, E, A, \eta, \alpha, \lambda)$-structure and a compatible metric g, the structure being induced from the natural structure on the ambient Euclidean space. If ∇ denotes the Riemannian connexion of g, then for the sphere example the structure tensors satisfy

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