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# CONTINUITY OF MAPPINGS OF VECTOR LATTICES WITH NORMS AND SEMINORMS

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Dirichlet mappings were introduced in [4] and further studied in [1], [5], and [7]. In the present paper we take up Dirichlet $p$-mappings and give a continuity property which will be of fundamental importance in the problem of characterizing these mappings geometrically.

In §1 we establish a continuity theorem, a generalization of [1], on isomorphisms of vector lattices with norms and seminorms. This theorem has interest in its own right and is the main content of the present paper. As an application we obtain in §2 the norm continuity of Dirichlet $p$-mappings. A detailed study of these mappings and their geometric characterization will appear elsewhere.

## §1. A general continuity theorem.

**1.** Let $X$ be a *normed vector lattice* over the real number field $\boldsymbol{R}$ such that $\boldsymbol{R}\subset X$, $\|x\|_X\leqq 1$ implies $|x|\leqq 1$, and

$$\alpha\leqq\|(\alpha\vee x)\wedge\beta\|_X\leqq\beta \tag{1}$$

for every $\alpha, \beta\in\boldsymbol{R}$ with $0\leqq\alpha\leqq\beta$ and every $x\in X$.

For a fixed number $p>1$ we consider a seminorm $q_X(\cdot)$ in $X$ with the properties

$$q_X(\alpha)=0 \tag{2}$$

for every $\alpha\in\boldsymbol{R}$,

$$q_X^p(x\wedge\alpha)+q_X^p(x\vee\alpha)=q_X^p(x) \tag{3}$$

for every $x\in X$ and every $\alpha\in\boldsymbol{R}$,

$$\lim_{\alpha'\uparrow\alpha,\beta'\downarrow\beta} q_X((\alpha'\vee x)\wedge\beta')=q_X((\alpha\vee x)\wedge\beta) \tag{4}$$

for every $x\in X$ and $\alpha, \beta\in\boldsymbol{R}$ with $\alpha\leqq\beta$.

A third norm $|||\cdot|||_X$ in $X$ is given by

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