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ON THE INFLUENCE OF A CONFORMAL KILLING TENSOR ON THE REDUCIBILITY OF COMPACT RIEMANNIAN SPACES

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§0. Let M be an n-dimensional Riemannian space whose metric tensor is given by g_{ab}.[1] A contravariant vector field v^a is called an infinitesimal conformal transformation or a conformal Killing vector if there exists a scalar function ρ such that

$$\nabla_a v_b + \nabla_b v_a = 2\rho g_{ab},$$

where $v_a = g_{ab} v^b$ and ∇_a means the covariant derivation with respect to the Riemannian connection. Especially, a conformal Killing vector v^a is called an infinitesimal isometry or a Killing vector if $\rho = 0$. In a compact reducible Riemannian space, the following theorem is well known.

THEOREM (Tachibana [1][2]). *In a compact reducible Riemannian space, an infinitesimal conformal transformation is an infinitesimal isometry.*

On the other hand, as a generalization of a conformal Killing vector, Kashiwada [3] has defined a conformal Killing tensor, that is, a skew-symmetric tensor $u_{a_1 \cdots a_r}$ is called a conformal Killing tensor of degree r if there exists a skew-symmetric tensor $\rho_{a_1 \cdots a_{r-1}}$ such that

$$(0.1) \qquad \nabla_c u_{a_1 \cdots a_r} + \nabla_{a_1} u_{c a_2 \cdots a_r} = 2\rho_{a_2 \cdots a_r} g_{c a_1} - \sum_{i=2}^{r} (-1)^i (\rho_{a_1 \cdots \hat{a}_i \cdots a_r} g_{c a_i} + \rho_{c a_2 \cdots \hat{a}_i \cdots a_r} g_{a_1 a_i}),$$

where $\hat{a}_i$ means that a_i is omitted. This $\rho_{a_1 \cdots a_{r-1}}$ is called the associated tensor of $u_{a_1 \cdots a_r}$. Especially, $u_{a_1 \cdots a_r}$ is called a Killing tensor if $\rho_{a_1 \cdots a_{r-1}} = 0$.

The purpose of the paper is to discuss the relation between the existence of a conformal Killing tensor and the reducibility of compact Riemannian spaces as a generalization of the above theorem.

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1) Indices $a, b, c, \cdots$ run over $1, \cdots, n$.

2) See the bibliography at the end of the paper.