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THE ASYMPTOTIC DISTRIBUTION OF INFORMATION PER UNIT COST CONCERNING A LINEAR HYPOTHESIS FOR MEANS OF GIVEN TWO NORMAL POPULATIONS

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§ 1. Introduction.

On the sequential design problem Chernoff [1] has studied a sequential testing problem concerned with composit hypothesis. In his paper he has shown an essential and simple example in which he treated two mutually independent Bernoulli trials T_1 and T_2. If we denote the probability of success of trial T_1 with p_1 and the probability of success of the trial T_2 with p_2 and the hypothesis $p_1=p_2$ with H_0 and $p_1 \neq p_2$ with H_1, the subject which he treated was sequential test of the hypothesis H_0. He has given a selecting way of trial at each step definitely considering the results of preceding observations. More precisely, the procedure is deterministically given step by step comparing with Kullback-Leibler (K-L) informations of T_1 and T_2.

We have studied in [2] the asymptotic behavior of the sum of informations which discriminate the hypotheses H_0 and H_1 gained between first and n-th step under the procedure using K-L information deterministically at each step.

In our paper [4], for given finite number of populations E_i $(i=1, \cdots, k)$ which has a distribution of exponential type with one dimensional parameter θ_i $(i=1, \cdots, k)$ respectively, we had treated the sequential testing problem with respect to the given hypothesis $\mu \cdot \theta = p$, concerning unknown parameters $\theta_1, \cdots, \theta_k$, in k dimensional $(\theta_1, \cdots, \theta_k)$ space. The distribution of i-th population E_i $(i=1, \cdots, k)$ was restricted by an exponential type introduced by S. Kullback. We have given in [2], [3] a cost optimal procedure $\mathcal{P}$ selecting the populations and in [4] the equivalent randomized procedure $\mathcal{P}^*$ the limiting property of the logarithm of the likelihood ratio per unit cost concerning the hypothesis $\mu \cdot \theta = p$ of our unknown k dimensional parameter $\theta=(\theta_1, \cdots, \theta_k)$.

We have specially had some interests on the asymptotic property of the deterministic procedure. Given two trials T_1 and T_2 each of which has a normal distribution with mean m_1 and m_2 and variance σ_1^2 and σ_2^2 respectively, then the hypothesis H_0 becomes $m_1=m_2$ and H_1 becomes $m_1 \neq m_2$ analogously. In this model using the deterministic procedure which compares with K-L informations of the given trials T_1 and T_2, the selecting ratio of T_1 and T_2 has strait convergence property to the optimal ratio as given in [3] and [4] as "special example".

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