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ON THE TOTAL ABSOLUTE CURVATURE OF MANIFOLDS IMMERSED IN RIEMANNIAN MANIFOLD, II[1)]

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In [3], [4] and [8], Chern, Lashof, Kuiper and Ōtsuki studied the total absolute curvature of an oriented compact manifold immersed in a euclidean space, and obtained some interesting results.

In [11], Willmore and Saleemi defined the total absolute curvature for compact oriented manifolds immersed in riemannian manifolds. In [1], the author used the Levi-Civita parallelism to define the total absolute curvature of compact manifolds immersed in a simply-connected riemannian manifold with non-positive sectional curvature, and proved that many results due to Chern-Lashof, Kuiper hold. In 1967, Kuiper [5] proposed to study the total absolute curvature for the surfaces immersed in euclidean 3-sphere.

In this present paper, we consider the total absolute curvature of manifolds immersed in arbitrary riemannian manifold, in particular, the surfaces in real space forms.

1. Preliminaries.

In the following, we assume throughout that M^n is an n-dimensional manifold, and Y^{n+N} is an oriented riemannian manifold of dimension $n+N$.

Let

$$f\colon\ M^n \to Y^{n+N} \tag{1}$$

be an immersion of M^n into Y^{n+N}.

In the following, by a frame $x, e_1, \cdots, e_{n+N}$ in Y^{n+N} we mean a point x and an ordered set of mutually perpendicular tangent unit vectors $e_1, \cdots, e_{n+N}$ at x, such that their orientation is coherent with that of Y^{n+N}. Unless otherwise stated, we agree on the following ranges of the indices:

$$1 \leqq i, j, k \leqq n, \qquad n+1 \leqq r, s, t \leqq n+N, \qquad 1 \leqq A, B, C \leqq n+N. \tag{2}$$

Let $F(Y^{n+N})$ be the bundle of the frames on Y^{n+N}. In $F(Y^{n+N})$, we introduce the linear differential forms θ_A, θ_{AB} by the equations:

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