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MINIMAL IMMERSIONS OF COMPACT RIEMANNIAN MANIFOLDS IN COMPLETE AND NON-COMPACT RIEMANNIAN MANIFOLDS

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§ 0. Introduction.

As is well known a closed surface in a Euclidian 3-space has at least one point where Gaussian curvature is positive, and hence no closed minimal surface exists in a Euclidean 3-space. This result was generalized by Myers [3] to higher dimensions and certain Riemannian manifolds. One of his results in [3] states that there exists no closed minimal hypersurface in a complete and simply connected Riemannian manifold of non-positive curvature. One of the essential ideals for proof of the theorem is "local concavity". This idea was used by Tompkins [4] in order to investigate the lower bound of the dimension of the Euclidean space in which a compact and flat Riemannian manifold can be immersed isometrically.

In this paper, we shall observe the idea of Tompkins from another point of view. We shall find certain property of second fundamental form of a compact Riemannian manifold immersed in a complete and non-compact Riemannian manifold of non-negative curvature. We shall prove the existence of a point and a unit normal vector e at the point on a compact Riemannian manifold immersed in a complete and non-compact Riemannian manifold of non-negative (positive) curvature at which the eigenvalues of the second fundamental form with respect to e are all non-negative (positive). Our main results obtained in the present paper will state as follows.

THEOREM A. *A compact Riemannian manifold N of dimension n cannot be immersed minimally in an $(n+1)$-dimensional complete and non-compact Riemannian manifold of positive Ricci curvature.*

THEOREM B. *A compact Riemannian manifold N of dimension n cannot be immersed minimally in an $(n+m)$-dimensional complete and non-compact Riemannian manifold of positive curvature.*

Our essential tool for the proofs of the results is quite analogous as the local concavity except the point of view. The compactness of an immersed Riemannian manifold N in a complete and non-compact Riemannian manifold M ensures the existence of a point on N which is the nearest to the point at infinity. If the

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