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INTEGRAL FORMULAS FOR SUBMANIFOLDS OF CODIMENSION 2 AND THEIR APPLICATIONS

BY KENTARO YANO AND MASAFUMI OKUMURA

§ 1. Introduction

Various integral formulas for hypersurfaces of a Riemannian manifold have been found and applied to the study of closed hypersurfaces with constant mean curvature.

Integral formulas for submanifolds of codimension greater than 1 was first obtained by Okumura [6] for the case of submanifolds of codimension 2 of an odd dimensional sphere. He made use of the natural contact structure of the odd dimensional sphere. Integral formulas for general submanifolds of a Riemannian manifold have been obtained by Katsurada [1], [2], [3], Kôjyô [2], Nagai [3], [4], and Yano [9].

In a recent paper [7], Okumura obtained integral formulas for a submanifold of codimension 2, invariant under the curvature transformation, of a Riemannian manifold admitting an infinitesimal conformal transformation and used them to prove that, under certain conditions, the submanifold in question is totally umbilical.

In the present paper, we study a problem similar to that treated in [7]. In [7], the ambient Riemannian manifold was supposed to admit an infinitesimal conformal transformation, but in this paper, we assume instead that there exists a vector field along the submanifold whose covariant differential is proportional to the displacement. We do not assume that the submanifold is invariant under the curvature transformation but instead we put a condition on the integral of a quantity depending on the curvature.

We moreover study the case in which the ambient Riemannian manifold admits a scalar function v such that $\nabla_j \nabla_i v = f(v) g_{ji}$ and prove that the submanifold satisfying certain conditions is isometric to a sphere by a method used in [8].

§ 2. Submanifolds of codimension 2.

We consider an $(n+2)$-dimensional orientable Riemannian manifold M^{n+2} of differentiability class C^∞ covered by a system of coordinate neighborhoods $\{U; x^h\}$, where and in the sequel the indices $h, i, j, \cdots$ run over the range $\{1, 2, \cdots, n, n+1, n+2\}$. We denote by g_{ji}, $\left\{ {}_j{}^h{}_i \right\}$, ∇_i, and $K_{kji}{}^h$, the metric tensor, the Christoffel symbols formed

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