KŌDAI MATH. SEM. RFP.
21 (1969), 335—349

INTEGRAL FORMULAS FOR CLOSED HYPERSURFACES

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§ 0. Introduction.

Liebmann [7] and Süss [9] proved that only ovaloid with constant mean curvature of a Euclidean space is a sphere. To prove this, we need an integral formula of Minkowski. So that to generalize the theorem above to the case of closed hypersurfaces of a Riemannian manifold, we must first of all obtain an integral formula for closed hypersurfaces of a Riemannian manifold. In the case of hypersurfaces of a Euclidean space, the so-called position vector plays an important rôle. So, to obtain the integral formulas for closed hypersurfaces in a Riemannian manifold, we assume the existence of a certain vector field, for example, a conformal Killing vector field or a concurrent vector field in a Riemannian manifold.

The study in this direction has been done by Hsiung [2], [3], [4], Katsurada [5], [6], Shahin [8], Tani [10] and Yano [11], [12].

Let V be a closed and orientable hypersurface of an $(n+1)$-dimensional Euclidean space E and denote by g, h and M_l the first fundamental tensor, the second fundamental tensor and the l-th mean curvature of the hypersurface respectively. Let $X(x^h)$ be the position vector from a fixed point O in E to a point P on the hypersurface V, where x^h are parameters on the hypersurface and N the unit normal to the hypersurface, and put $\alpha=X\cdot N$, $X_i=\partial X/\partial x^i$ and $z_i=X\cdot X_i$.

Shahin [8] recently proved the integral formulas

$$m\int_V \alpha^{m-1}h_{ji}z^j z^i dV-n\int_V \alpha^m(1+\alpha M_1)dV=0,$$

$$m\int_V \alpha^{m-1}M_n g_{ji}z^j z^i dV-n\int_V \alpha^m(M_{n-1}+\alpha M_n)dV=0,$$

for an arbitrary m for which α^{m-1} and α^m have meaning, dV being the volume element of V.

These formulas generalize those of Chern [1], Hsiung [2], [3], [4] and Shahin [8].

The main purpose of the present paper is to obtain a series of integral formulas the first and the last of which are those given by Shahin and to generalize this to the case of hypersurfaces of a Riemannian manifold.

Received March 17, 1969.