KŌDAI MATH. SEM. REP.
21 (1969), 331—334

SURFACES OF CURVATURE $\lambda_N=0$ IN E^{2+N}

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1.[1),2)] In [3], Prof. Ōtsuki introduced some kinds of curvature, $\lambda_1, \lambda_2, \cdots, \lambda_N$, for surfaces in a $(2+N)$-dimensional Euclidean space E^{2+N}. These curvatures play a main rôle for the surfaces in higher dimensional Euclidean space.

In [5], Shiohama proved that a complete, oriented surface M^2 in E^{2+N} with the curvatures $\lambda_1=\lambda_2=\cdots=\lambda_N=0$ is a cylinder.

In this note, we shall prove the following theorem:

THEOREM 1. *Let f: $M^2 \to E^{2+N}$ $(N \geqq 2)$ be an immersion of a compact, oriented surface M^2 in a $(2+N)$-dimensional Euclidean space E^{2+N}. Then*

(I) *The last curvature $\lambda_N=0$ if and only if M^2 is imbedded as a convex surface in a 3-dimensional linear subspace of E^{2+N}, and*

(II) *The first curvature $\lambda_1=a=constant$ and the last curvature $\lambda_N=0$ if and only if M^2 is imbedded as a sphere in a 3-dimensional linear subspace of E^{2+N} with radius $1/\sqrt{a}$.*

2. Lemmas. In order to prove Theorem 1, we first prove the following two lemmas.

LEMMA 1. *Let f: $M^2 \to E^{2+N}$ be an immersion given as in Theorem 1. Then the last curvature $\lambda_N \geqq 0$ if and only if M^2 is imbedded as a convex surface in a 3-dimensional linear subspace of E^{2+N}.*

Proof. Let f: $M^2 \to E^{2+N}$ be an immersion given as in Theorem 1, and let $(p, e_1, e_2, \cdots, e_{2+N})$ be a Frenet-frame in the sense of Ōtsuki [2], then we have the following:

$$dp=\omega_1 e_1+\omega_2 e_2, \tag{2.1}$$

$$de_A=\sum_B \omega_{AB} e_B, \qquad \omega_{AB}+\omega_{BA}=0, \tag{2.2}$$

$$\omega_{ir}=\sum_r A_{rij}\omega_j, \qquad A_{rij}=A_{rji}, \tag{2.3}$$

$$\omega_{ir}\wedge\omega_{2r}=\lambda_{r-2}\omega_1\wedge\omega_2 \qquad \lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_N, \tag{2.4}$$

$$G(p)=\sum_r \lambda_{r-2}(p), \tag{2.5}$$

$$A, B=1, \cdots, 2+N, \quad r=3, \cdots, 2+N, \quad i, j=1, 2,$$

Received February 10, 1969.

1) We follow the notations in [3].

2) This work was supported in part by the SDF at University of Notre Dame, 1968–1970.