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TENSOR FIELDS AND CONNECTIONS IN CROSS-SECTIONS IN THE TANGENT BUNDLE OF ORDER 2

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The prolongations of tensor fields and connections given in a differentiable manifold M to its tangent bundle $T(M)$ have been studied in [1], [2], [5], [7]. If a vector field V is given in M, V determines a cross-section in $T(M)$ which is as an n-dimensional submanifold in $T(M)$. Yano [3] has recently studied the behavior of the prolongations of tensor fields and connections to $T(M)$ on the cross-sections determined by a vector field in M. On the other hand, the prolongations of tensor fields and connections in M to its tangent bundle $T_2(M)$ of order 2 are studied in [6]. If a vector field V is given in M, V determines a cross-section in $T_2(M)$. The main purpose of the present paper is to study the behavior of the prolongations of tensor fields and connections in M to $T_2(M)$ on the cross-section determined by a vector field in M.

In § 1 we first recall properties of the prolongations of tensor fields and connections in M to $T_2(M)$. In § 2 we study the cross-sections determined in $T_2(M)$ by vector fields given in M. § 3 will be devoted to the study of the prolongations of tensor fields given in M to $T_2(M)$ along the cross-sections and § 4 will be devoted to the study of the prolongations of connections given in M to $T_2(M)$ along the cross-sections.

§ 1. Prolongations of tensor fields and linear connections to the tangent bundle of order 2.

We shall recall, for the later use, some properties of the tangent bundle $T_2(M)$ of order 2 over a differentiable manifold M of dimension n, and those of prolongations of tensor fields and linear connections in M to $T_2(M)$ (cf. [6]).

The tangent bundle $T_2(M)$ of order 2 is the space of equivalence classes of mappings from the real line R into M, the equivalence relation being defined as follows: we say that two mappings F and G are equivalent to each other if, in a coordinate neighborhood U, they satisfy the conditions

$$F(0)=G(0)=p, \qquad \frac{dF^h}{dt}(0)=\frac{dG^h}{dt}(0), \qquad \frac{d^2F^h}{dt^2}(0)=\frac{d^2G^h}{dt^2}(0),$$

where $F^h(t)$ and $G^h(t)$ are the coordinates of $F(t)$ and $G(t)$ in U respectively. This

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