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GENERALIZATIONS OF THE CONNECTION OF TZITZÉICA

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Dobrescu [1] has recently studied what he calls the connection of Tzitzéica [4] on hypersurfaces in a Euclidean space.

The main purpose of the present paper is to define the connection of Tzitzéica on hypersurfaces in an affinely connected manifold along which a torse-forming [6] or a concurrent vector field [5] is given and to study the properties of the connection of Tzitzéica thus defined.

§ 1. The connection of Tzitzéica on a hypersurface in a centro-affine space.

Let A^n be an n-dimensional centro-affine space ($n \geqq 3$), that is, an affine space in which a point O is specified. Then any point P in A^n is represented by the so-called position vector $X=\overrightarrow{\mathrm{OP}}$. This means that with every point P of A^n, there is associated a vector X.

We now assume that there is given a hypersurface V^{n-1} in A^n and denote by

$$X=X(u^1, u^2, \cdots, u^{n-1})$$

the parametric representation of V^{n-1}, where (u^a) $(a, b, c, \cdots=1, 2, 3, \cdots, n-1)$ are local parameters on V^{n-1} such that vectors

$$X_b=\partial_b X$$

tangent to V^{n-1} are linearly independent, ∂_b denoting the differential operators $\partial_b=\partial/\partial u^b$.

We assume that the vector X at P on V^{n-1} is never tangent to V^{n-1}, that is, the vector X is linearly independent of X_b.

We then have, for the vectors $\partial_c X_b$, the equations of the form

$$\partial_c X_b=\Gamma_c{}^a{}_b X_a+h_{cb}X, \tag{1.1}$$

where $\Gamma_c{}^a{}_b$, symmetric in c and b, define an affine connection on V^{n-1} called the connection of Tzitéica [1], [4] and h_{cb}, symmetric in c and b, define a tensor field on V^{n-1} called the second fundamental tensor.

The equations (1.1) are so-called equations of Gauss for the V^{n-1}, the pseudo-

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