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CONSTRUCTION OF BRANCHING MARKOV PROCESSES WITH AGE AND SIGN*

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It is a familiar fact in the theory of Markov processes that solutions for a wide class of *linear* parabeolic and elliptic equations can be investigated in terms of Markov processes (e.g. Dynkin [3] and Ito-McKean [8]). On the other hand it was known that a class of *semi-linear* parabolic equations plays an important rôle in the theory of branching processes (e.g. Bartlett [2], Harris [5], Moyal [15], and Skorohod [21]). The mathematical structure that reveals the mechanism of how this non-linearity appears in the theory of Markov processes should, therefore, be investigated systematically. Attempts in this direction were recently performed in several articles, especially, in Moyal [13], [14], [15], Skorohod [21], and Ikeda-Nagasawa-Watanabe [6], [7]. A branching Markov process X_t is defined as a strong Markov process on a large state space $\hat{\boldsymbol{S}}=\bigcup_{n=0}^{\infty} S^n \cup \{\Delta\}$ having the following branching property[1]

$$T_t\hat{f}(x)=\widehat{(T_t\hat{f})|_S}(x), \qquad x\in\hat{\boldsymbol{S}}, \tag{1}$$

where T_t is the semi-group of the "large" Markov process on $\hat{\boldsymbol{S}}$ and $\hat{f}$ is a function on $\hat{\boldsymbol{S}}$ defined by

$$\hat{f}(x)=\begin{cases} 1 \quad , & \text{if } \ x\in S^0, \\ \prod_{i=1}^{n} f(x_i), & \text{if } \ x=(x_1, x_2, \cdots, x_n)\in S^n, \\ 0 \quad , & \text{if } \ x=\Delta, \end{cases}$$

where f is a measurable function on S with $\|f\|=\sup_{x\in S}|f(x)|\leqq 1$. Then

$$u(t,x)=T_t\hat{f}(x), \qquad x\in S,$$

is the (minimal)[2] solution of a non-linear integral equation:

$$u(t,x)=T_t^0 f(x)+\int_0^t K(x, dsdy)F[y, u(t-s, \cdot)], \qquad x\in S, \tag{2}$$

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1) Definition of the notation appearing in the following will be found in §1.
2) This is taken to mean that if $f\geqq 0$, $u(t,x)=T_t\hat{f}(x)$ is the minimal solution of (2).