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ON THE ABSOLUTE NÖRLUND SUMMABILITY OF A FOURIER SERIES AND ITS CONJUGATE SERIES

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1. Definitions and notations. Let $\sum a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of constants, real or complex, and let us write $P_n=p_0+p_1+\cdots+p_n$, $P_{-1}=p_{-1}=0$. The sequence-to-sequence transformation:

$$t_n=\sum_{\nu=0}^{n} p_{n-\nu}s_\nu/P_n, \qquad (P_n\neq 0) \tag{1. 1}$$

defines the sequence $\{t_n\}$ of Nörlund means[1] of $\{s_n\}$, generated by the sequence of coefficients $\{p_n\}$. If $\{t_n\}\in BV$, i.e., $\sum_n|t_n-t_{n-1}|<\infty$,[2] we say that $\sum a_n$ or $\{s_n\}$ is summable $|N, p_n|$.[3]

In the special case in which $p_n=\binom{n+\alpha-1}{\alpha-1}$, $\alpha>-1$, the (N, p_n) mean reduces to familiar (C, α) mean.

Let $f(t)$ be a periodic function with period 2π and integrable in the Lebesgue sense over $(-\pi, \pi)$ and let the Fourier series of $f(t)$ be

$$\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=1}^{\infty}A_n(t). \tag{1. 2}$$

Then the conjugate series of (1. 2) is

$$\sum_{n=1}^{\infty}(b_n\cos nt-a_n\sin nt)=\sum_{n=1}^{\infty}B_n(t). \tag{1. 3}$$

We write throughout:

$$\varphi(t)=\frac{1}{2}\{f(x+t)+f(x-t)\}, \qquad \psi(t)=\frac{1}{2}\{f(x+t)-f(x-t)\};$$

$$R_n=(n+1)p_n/P_n; \qquad S_n=\sum_{\nu=0}^{n}(\nu+1)^{-1}P_\nu/P_n;$$

$$P_n^*=\sum_{\nu=0}^{n}|p_\nu|; \qquad S_n^*=\sum_{\nu=0}^{n}(\nu+1)^{-1}|P_\nu|/|P_n|;$$

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1) Nörlund [6]. See also Woronoi [14].

2) Similarly by '$F(x)\in BV(a, b)$', we mean that $F(x)$ is a function of bounded variation in the interval (a, b) and '$\{\mu_n\}\in B$' means that $\{\mu_n\}$ is a bounded sequence.

3) Mears [5].