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CAPACITABILITY AND EXTREMAL RADIUS

BY NOBUYUKI SUITA

1. Introduction. Let Ω be a plane region and let α be its preassigned boundary component. In a previous paper of these reports [5] we constructed a circular and radial slit disc mapping of the region with respect to a partition, denoted by (α, A, B), of its boundary. In this construction, the coincidence and finiteness of the radii $\overline{R}(A)$ and $\underline{R}(B)$ defined below, were assumed. Then the following problem will arise: *When do the quantities $\overline{R}(A)$ and $\underline{R}(B)$ coincide?* We shall give an answer to this problem, making use of Choquet's theory of capacities [2]. The answer is as follows: Let the set A be generated by the Souslin operation from the class of closed set of boundary components in the Stoïlow compactification of the region less α. Then $\overline{R}(A)$ is equal to $\underline{R}(B)$.

We can see, as its consequence, that the univalent functions which correspond to a minimal sequence of $\overline{R}(A)$ and a maximal sequence of $\underline{R}(B)$, constructed in no. **4** are really circular and radial slit disc mappings.

So far as the construction of capacity functions concerns these results holds on open Riemann surfaces. The basic results for the partitions (α, A, B) in which A or B is closed were discussed by Marden and Rodin [3].

2. Preliminaries. Let Ω be a plane region which is not the extended plane. We denote by $\hat{\Omega}$ the Stoïlow compactification of Ω in which each boundary component is a point. Let α be a preassigned boundary component and let (α, A, B) denote a partition of the boundary $\partial\Omega=\hat{\Omega}-\Omega$.

A curve c is a continuous image of the closed interval $[0, 1]$ into $\hat{\Omega}$. It is said to be locally rectifiable, if so is every component of $\Omega\cap c$. All quantities such as length, integral etc. are defined about the restriction of c on Ω.

Let a be a point of Ω. We denote by $\Gamma(\alpha, A, B)$ and $X(\alpha, A, B)$ the families of locally rectifiable curves separating α from a within $\hat{\Omega}-A$ and joining them within $\hat{\Omega}-B$ respectively. Let $\Gamma_q(\alpha, A, B)$ and $X_q(\alpha, A, B)$ denote the families in the difinitions of which the point a is replaced by a compact disc $|z-a|\leqq q$ in Ω. We define two quantities by

$$\log R_1=\lim_{q\to 0}(2\pi \bmod \Gamma_q(\alpha, A, B)+\log q) \tag{1}$$

and

$$\log R_2=\lim_{q\to 0}(2\pi\lambda(X_q(\alpha, A, B))+\log q), \tag{2}$$

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