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ON LOCAL MAXIMALITY FOR THE COEFFICIENTS a_6 and a_8

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1. In our previous papers [1], [2], [3] we proved the local maximality of $\Re a_6$ and $\Re a_8$ at the Koebe function $z/(1-z)^2$. In this note we shall prove the local maximality of $|a_6|$ and $|a_8|$ at the Koebe function $z/(1-e^{i\theta}z)^2$, that is, the following theorems.

THEOREM 1. *Let $f(z)$ be a normalized regular function univalent in the unit circle*

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n.$$

Then there is a positive constant ε such that $|a_6|\leqq 6$ holds for $0\leqq 2-|a_2|\leqq\varepsilon$. Equality occurs only for the Koebe function $z/(1-e^{i\theta}z)^2$.

THEOREM 2. *$|a_8|\leqq 8$ holds for $0\leqq 2-|a_2|\leqq\varepsilon$. Equality occurs only for the Koebe function $z/(1-e^{i\theta}z)^2$.*

In the sequel we shall use the same notations as in [1], [2], [3]. Further we put $p=2-x$, $x'=kp$.

2. Proof of theorem 1. By the well-known rotation it is sufficient to prove that

$$\text{(A)} \qquad \Re a_6 < 6$$

for $0\leqq 2-|a_2|\leqq\varepsilon$, $|\arg a_2|\leqq\pi/4$, unless $a_2=2$. Then we can use our earlier result in [1], [3]:

$$\Re a_6\leqq 6-A(2-\Re a_2), \qquad A>0$$

holds for $0\leqq 2-\Re a_2\leqq\varepsilon_1$. Here equality occurs only for the Koebe function $z/(1-z)^2$.

Hence there are positive constants ε_2 and δ' such that $\Re a_6<6$ for $0\leqq 2-|a_2|\leqq\varepsilon_2$, $|\arg a_2|\leqq\delta'$, unless $a_2=2$. Hence we may assume that $0<\tan\delta'=\delta\leqq|k|\leqq 1$.

By Grunsky's inequality $|b_{55}|\leqq 1$ we have

$$\left| a_6-2a_2a_5-3a_3a_4+4a_4a_2^2+\frac{21}{4}a_2a_3^2-\frac{59}{8}a_3a_2^3+\frac{689}{320}a_2^5\right|\leqq\frac{2}{5}.$$

By taking the real part we have

$$\Re a_6\leqq\frac{2}{5}+\Re\left\{2(p+ix')(\xi+i\xi')+3(y+iy')(\eta+i\eta')+\frac{5}{4}(p+ix')^2(\eta+i\eta')\right.$$

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