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ON CONTACT METRIC IMMERSION

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Introduction. The theory of complex submanifolds in a complex manifold is one of the most fruitful aspects in the study of complex manifold. In fact, after Schouten and Yano [6] established the notion of so-called invariant submanifold of a complex manifold many beautiful theorems concerning this have been proved.

On the other hand Sasaki [4] established a differential geometric method to study a contact manifold and this permits us to study contact manifold by use of tensor calculus. Making use of Sasaki's method, Watanabe [7] and the present author [2, 3] studied some submanifolds of a contact manifold.

However, in their papers, they observed rather Riemannian structures of the submanifold than contact metric structures.

In this paper, the author tries to establish a theory of submanifold which is inherited a contact metric structure by the enveloping contact metric manifold.

In § 1 we give first of all the definition of contact metric manifold and in § 2 a summary of theory of submanifolds of codimension 2 in a Riemannian manifold. These two paragraphs are rather expository. After these preliminaries we give in § 3 some formulas in a submanifold of codimension 2 in a contact metric manifold.

In § 4 we define the notion of contact metric immersion of a manifold into a contact metric manifold of codimension 2 and show conditions for an immersion to be a contact metric one.

Further in this paragraph we study the relations between a contact metric immersion and an immersion which is called F-invariant one.

In § 5 we define the notion of normal contact immersion of a manifold into a normal contact manifold of codimension 2 and prove conditions for an immersion to be a normal contact one.

Finally in § 6 we show an example which is an umbilical submanifold in normal contact manifold but not a normal contact submanifold.

§ 1. Contact metric manifold.

A $(2n+1)$-dimensional differentiable manifold $\tilde{M}^{2n+1}$ is said to have a contact structure and called a contact manifold if there exists a 1-form $\tilde{\eta}$ on $\tilde{M}^{2n+1}$ such that

$$\tilde{\eta}\wedge(d\tilde{\eta})^n\neq 0 \tag{1.1}$$

everywhere on $\tilde{M}^{2n+1}$ where $d\tilde{\eta}$ is the exterior derivative of $\tilde{\eta}$ and the symbol $\wedge$

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