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ON SOME CONFORMAL EQUIVALENCE CONDITIONS OF COMPACT RIEMANN SURFACES

BY YASUSHI MIYAHARA

The purpose of this paper is to obtain some conditions for two compact Riemann surfaces to be conformally equivalent. We shall mention our results by use of the Douglas-Dirichlet functional and harmonic mappings.

Let R and S be compact Riemann surfaces of genus g, and let $\eta=\rho(w)|dw|^2$ be a conformal metric on S, where $\rho(w)$ is positive and continuous with respect to each local parameter w on S. We call η a *normalized conformal metric* on S, if it satisfies

$$\iint_S \rho(w)dudv=1.$$

Let f be an orientation-preserving homeomorphism of R onto S. We assume that f is L_2-derivable, that is, $w=f(z)$ has generalized partial derivatives which are square integrable, where $w=f(z)$ is a local representation of f for local parameters z and w on R and S, respectively. Since f is orientation-preserving, we have

$$\left|\frac{\partial f}{\partial z}\right|^2-\left|\frac{\partial f}{\partial \bar{z}}\right|^2\geqq 0$$

almost everywhere in each parametric disk on R. Furthermore, it is known that f is a measurable mapping, and

$$\text{mes } f(E)=\iint_E\left(\left|\frac{\partial f}{\partial z}\right|^2-\left|\frac{\partial f}{\partial \bar{z}}\right|^2\right)dxdy$$

for any measurable set E on R (cf. [3]). The integral

$$I_\eta[f]=\iint_R \rho(f(z))\left(\left|\frac{\partial f}{\partial z}\right|^2+\left|\frac{\partial f}{\partial \bar{z}}\right|^2\right)dxdy$$

is called the *Douglas-Dirichlet integral*. If $\eta=\rho(w)|dw|^2$ is a normalized conformal metric on S, we have

$$I_\eta[f]-1=2\iint_R \rho(f(z))\left|\frac{\partial f}{\partial \bar{z}}\right|^2 dxdy,$$

since

$$\iint_R \rho(f(z))\left(\left|\frac{\partial f}{\partial z}\right|^2-\left|\frac{\partial f}{\partial \bar{z}}\right|^2\right)dxdy=1.$$

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