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AN ASPECT OF LOGISTIC LAW

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1. Introduction.

In social or economic phenomena, it is empirically known that certain variables such as human populations increase monotonously or *grow* with time, obeying a certain law, the logistic law of growth. Few basic considerations, however, have been made from the theoretical point of view. (See for example Feller [1]). For investigating the logistic law more firmly it seems to the authors that the growing variables should be taken into account in correlation with suitable parameters.

Suggested by what we have obtained while dealing with actual statistical data, we shall give, in this paper, introducing an important notion of *translatability*, some concrete bases for the use of logistic law, which indicate a direction to the basic interpretation of the growing phenomena.

2. The logistic curve.

A variable $p(t)$, measured in an appropriate unit, will be called to obey logistic law of growth or simply logistic law, if

$$p(t)=\frac{1}{1+\alpha e^{-\beta t}}$$

for arbitrary positive constants α and β, where t $(-\infty<t<\infty)$ denotes time.

Note here that the growing variable $p(t)$ does not always mean a relative frequency or a probability. Note also that $p''(t_0)=0, p(t_0)=1/2$ for $t=t_0$ such that $\alpha e^{-\beta t_0}=1$.

3. A theoretical model.

Suppose there exists a parameter m, $m>0$, which has at any specified time t a correlation with a variable p, $p>0$, so that the relation between m and p can be represented by a regression line:

$$(1) \qquad p=a(t)m, \qquad 0<a(t)<1,$$

where $a(t)$ is a non-constant differentiable time function which is the gradient of the line at time t, and satisfies that $a(t_1)<a(t_2)$ if $t_1<t_2$.

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