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A REMARK ON ULTRAHYPERELLIPTIC SURFACES

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1. Let R be an ultrahyperelliptic surface of finite order and with $P(R)=4$. Let p denote its order, then R is defined by

$$y^2=(e^H-\gamma)(e^H-\delta), \qquad \gamma\delta(\gamma-\delta)\neq 0$$

with $H(z)=\sum_{n=1}^{p} h_n z^n$, $h_p\neq 0$. [3]. Let S be another ultrahyperelliptic surface of non-zero finite order, that is,

$$y^2=g(z)\equiv z^s \prod_{n=1}^{\infty} E\left(\frac{z}{a_n}, q\right),$$

where $E(u, q)$ is the Weierstrass prime factor

$$E(u, q)=(1-u)\exp\sum_{j=1}^{q}\frac{u^j}{j}$$

and every a_n is a simple zero of $g(z)$ and $s=0$ or 1.

Hiromi and Mutō [1] proved the following result: Assume there exists a non-trivial analytic mapping φ from R into S. Then $p=n\cdot r$, where r is the order of $g(z)$ and n is an integer.

The aim of the present paper is to prove the following

THEOREM. *If S is an ultrahyperelliptic surface of non-zero finite order into which there is a non-trivial analytic mapping from an ultrahyperelliptic surface R of finite order and with $P(R)=4$, then the order of S is a half of an integer.*

2. Proof of theorem. For our purpose we need our previous result in [4], which asserts the existence of two functions $h(z)$ and $f(z)$ such that $f(z)$ is meromorphic in $|z|<\infty$ and $h(z)$ is a polynomial of degree n in the present situation [1] satisfying

$$f(z)^2(e^H-\gamma)(e^H-\delta)=g\circ h(z).$$

Here put $h(z)=\sum_{m=0}^{n} h_m^* z^m$.

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