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FIBRED SPACES WITH INVARIANT RIEMANNIAN METRIC

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Introduction. In a previous paper [11], we developed the differential geometry of fibred spaces mainly from the point of view of affine geometry, that is, we studied the fibred spaces with invariant affine connection. We also studied the fibred spaces which can represent the manifolds with projective connection.

In the present paper, we would like to study the fibred spaces with invariant Riemannian metric. Since the Riemannian connection determined by the invariant Riemannian metric is also invariant, fibred spaces with invariant Riemannian metric are fibred spaces with invariant affine connection in the sense of our previous paper [11].

In § 1, we define fibred spaces with invariant Riemannian metric and recall some of fundamental concepts in fibred spaces with invariant affine connection. We study also induced metric, induced connection, second fundamental tensor, co-Gauss equations, co-Weingarten equations, and Nijenhuis tensor of the second fundamental tensor.

We develop in § 2 the tensor calculus in terms of local coordinates in a fibred space with invariant Riemannian metric and we study in § 3 some important formulas useful for discussions which follow. § 4 is devoted to the study of geodesics in the total space and in the base space.

We study in § 5 structure equations and curvatures. Starting from co-Gauss, co-Codazzi and co-Ricci equations, we derive relations between the curvature of the total space and that of the base space and prove propositions in which a Kähler or an almost Kähler structure appears. In § 6 we study some of interesting special cases.

In the last § 7, we study the case in which a fibred space with invariant Riemannian metric is a fibred space with K-contact structure.

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1) The number in brackets refers to the References at the end of the paper.