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CURVATURE-PRESERVING TRANSFORMATIONS OF K-CONTACT RIEMANNIAN MANIFOLDS

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Let M be a contact Riemannian manifold with a contact form η, the associated vector field ξ, (1, 1)-tensor field ϕ and the associated Riemannian metric g. If ξ is a Killing vector field, M is said to be a K-contact Riemannian manifold. Further, M is said to be normal, if ϕ satisfies the relation

$$(\nabla_X \phi)(Y)=g(X, Y)\xi-\eta(Y)X$$

for any vector fields X and Y on M, where ∇ is the covariant differentiation with respect to g.

Recently Okumura [2] got the following result:

(A) *In a normal contact Riemannian manifold, any curvature-preserving infinitesimal transformation is an infinitesimal isometry.*

On the other hand, Sakai [3] got the result:

(B) *Any affine transformation of a K-contact Riemannian manifold is an isometry.*

In this note, we prove the next theorem which covers the above (A) and (B):

THEOREM. *Let M, N be K-contact Riemannian manifolds, then any curvature-preserving transformation of M to N is an isometry.*

The proof of our theorem has similar aspect to that in [3]. In an m-dimensional K-contact Riemannian manifold we have

$$R_1(\xi, X)=(m-1)\eta(X), \tag{1}$$

$$R(X, \xi)\xi=-X+\eta(X)\xi \tag{2}$$

for any vector field X on M, where R_1 and R denote the Ricci curvature and Riemannian curvature tensor [1].

§ Proof of the theorem.

We denote the corresponding tensors in N by "$'$". Let φ be a curvature-preserving transformation of M to N and let x be an arbitrary point of M, and we put $y=\varphi x$. By X, Y, Z, W we denote vector fields on M. In any Riemannian manifold we have

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