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ASYMPTOTICALLY MOST INFORMATIVE PROCEDURE IN THE CASE OF EXPONENTIAL FAMILIES

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§1. Introduction.

Recently we showed the following fact in our paper [2]. We considered in [2] two binomial trials E_1, E_2 having unknown means p_1, p_2 respectively. And we have introduced the *notion of costs* such that we must pay costs c_1, c_2 to the observation of a result given by the trials E_1, E_2 respectively. In each step we are admitted to select one of the two trials E_1, E_2. Be continued the selections by some way we denoted the sequence of trials till n-th step as $E^{(1)}, \cdots, E^{(n)}$ and the sequence of costs till n-th step as $C^{(1)}, \cdots, C^{(n)}$. Of course we may select at i-th step $E^{(i)}$ from the two trials E_1, E_2 depending previous $i-1$ data $X_1, \cdots, X_{i-1}$ given by $E^{(1)}, \cdots, E^{(i-1)}$. A procedure $\mathfrak{L}$ was given in [2] such that the sum of information given by two dimensional likelihood ratio relative to the sum of costs till n-th step to discriminate $p_1>p_2$ or $p_1<p_2$ is asymptotically maximized. In [2] we assumed the unknown true two dimensional parameter (p_1, p_2) did not exist on the boundary $p_1=p_2$. In our another paper [3] we considered analogous model having two kinds of trials E_1, E_2 which are obeyed normal distributions with unknown means m_1, m_2 and known same variance σ^2 and costs c_1, c_2 respectively. Then analogous procedure $\mathfrak{L}$ is asymptotically optimal in the same sense described above. In [3] we noted that our procedure $\mathfrak{L}$ reduced to a policy which does not depending on previous n data $X_1, \cdots, X_n$ but only on sample sizes n_1 of E_1, n_2 of E_2 till n-th step. We have omitted the proof of the problem in [3] because we can easily get analogous proof.

In this paper we generalize these problems to k trials $E_1, \cdots, E_k$ having exponential distributions with one dimensional unknown parameter $\theta_1, \cdots, \theta_k$ respectively. That is, an observation X of E_j has a probability density function of exponential type in Kullback's sense [4] with one dimensional unknown parameter $\theta_j (j=1, \cdots, k)$ respectively. And we introduced the boundary π: $\mu\cdot\theta=p\,(\theta=(\theta_1, \cdots, \theta_k))$ as a hyperplane in k dimensional euclidean space where $\mu=(\mu_1, \cdots, \mu_k)$ is any fixed k dimensional unit vector having all non-zero components and p is any fixed non-negative number and $\mu\cdot\theta$ is the inner product of two vectors μ and θ. Moreover we use the *notion of costs* introduced by Kunisawa [6], as we used the notion in [2], [3], then we can get some information of θ_j by paying of cost $c_j (j=1, \cdots, k)$ respectively. Then we shall show analogously that under the generalized procedure $\mathfrak{L}^*$ given in the following Section 3 the sum of information relative to the sum of costs payed till n-th step to discriminate $\mu\cdot\theta$ larger than p or not is asymptotically

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