AN IMPROVEMENT OF A LIMIT THEOREM ON (J, X)-PROCESSES

BY HIROHISA HATORI AND TOSHIO MORI

1. Let $\boldsymbol{I}_r=\{1, 2, \cdots, r\}$ and $R=(-\infty, \infty)$, and let $\{(J_n, X_n);\ n=0, 1, 2, \cdots\}$ be a (J, X)-process with the state space $\boldsymbol{I}_r \times R$, or a two-dimensional stochastic process that statisfies $X_0 \equiv 0$, and

$$P\{J_n=k,\ X_n \leqq x \mid (J_0, X_0), \cdots, (J_{n-1}, X_{n-1})\}=Q_{J_{n-1},\ k}(x) \qquad \text{(a. s.)}$$

for all $(k, x) \in \boldsymbol{I}_r \times R$, where $\{Q_{jk}(\cdot);\ j, k=1, 2, \cdots, r\}$ is a family of non-decreasing functions defined on R such that $Q_{jk}(-\infty)=0$ for $j, k=1, 2, \cdots, r$, and $\sum_{k=1}^{r} Q_{jk}(+\infty)=1$ for $j=1, 2, \cdots, r$. Let I be the $r \times r$ identity matrix and let $P=(p_{jk})$ be the $r \times r$ matrix with elements $p_{jk} \overset{\text{def}}{=} Q_{jk}(+\infty)$. Throughout this paper we assume that there exists a positive integer m for which every element of the matrix P^m is positive. Then the equation

$$\det(I-zP)=0 \tag{1}$$

has the root $\alpha_0=1$ as a simple root and the remaining roots $\alpha_1, \cdots, \alpha_{k-1}$ are greater than 1 in absolute value. In the previous paper [1], we have proved a limit theorem concerning the (J, X)-process by assuming that (i) the polynomial $\det(I-zP)$ is of r-th degree and (ii) all roots of (1) are simple. In this paper we shall prove the conclusion of this theorem without these two assumptions.

REMARK. In the previous paper, we derived a sequence $\{Y_n\}$ of random variables from the (J, X)-process and proved a theorem for the sequence $\{Y_n\}$. However, it is sufficient to consider only the sequence $\{X_n\}$ of random variables in place of the sequence $\{Y_n\}$. Because we may show that the stochastic process $\{(J_n, Y_n);\ n \geqq 0\}$ is also a (J, X)-process. In what follows, we shall prove this fact. Since we have from the definition of $\{Y_n\}$ that

$$P\{Y_1 \leqq y_1, \cdots, Y_n \leqq y_n \mid J_0=k_0, \cdots, J_n=k_n\}$$

$$=\int \cdots \int P\{R_1(J_0, J_1, X_1) \leqq y_1, \cdots, R_n(J_{n-1}, J_n, X_n) \leqq y_n \mid J_0=k_0, \cdots, J_n=k_n, X_1=x_1, \cdots, X_n=x_n\}$$

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