f-STRUCTURES INDUCED ON SUBMANIFOLDS IN SPACES, ALMOST HERMITIAN OR KAEHLERIAN

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Introduction.

It is known that, if the tangent space of a submanifold of an almost complex space is invariant by the almost complex structure, the submanifold admits an almost complex structure. As a generalization of an almost complex structure, Yano [14] has introduced the concept of an f-structure in a differentiable manifold.

The purpose of this paper is to show that a submanifold of an almost complex space admits an f-structure under certain conditions and to study the induced f-structure.

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1. f-structures [14], [15], [17].

Let M^n be an n-dimensional connected differentiable manifold of class C^∞ and $\{\eta^c\}$ local coordinates. If there exists a non-vanishing tensor field f of type (1, 1) and of class C^∞ satisfying

$$f^3+f=0, \tag{1.1}$$

and the rank of f is constant everywhere and is equal to s, then we call[1] such a structure an f-structure of rank s. We put

$$l=-f^2, \qquad m=f^2+1, \tag{1.2}$$

where 1 denotes the unit tensor, then we have

$$l+m=1, \qquad l^2=l, \qquad m^2=m, \qquad lm=ml=0. \tag{1.3}$$

These equations mean that the operators l and m applied to the tangent space at each point of the manifold are complementary projection operators and there exist complementary distributions L and M corresponding to the operators l and m respectively. Then the distribution L is s-dimensional and M is $(n-s)$-dimensional. Further we get

$$\begin{aligned} fl=lf=f, &\qquad fm=mf=0, \\ f^2l=-l, &\qquad f^2m=0. \end{aligned} \tag{1.4}$$

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1) Yano [14].