ON THE EXISTENCE OF ANALYTIC MAPPINGS

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§1. Let R and S be two Riemann surfaces. When are there any analytic mappings from R into S? This would be one of the most important problems. However it is very difficult to establish a perfect condition for the existence of analytic mappings between two arbitrary Riemann surfaces.

Let R and S be two ultrahyperelliptic surfaces defined by two equations $y^2=G(z)$ and $u^2=g(w)$, respectively, where G and g are two entire functions having no zero other than an infinite number of simple zeros. Then Ozawa [1], [2], [3], [4] obtained a perfect condition for the existence of analytic mappings from R into S and several other interesting results.

In the first place (§2∼§5) we shall discuss the Riemann surfaces defined in the following manner: Let R_3 and S_3 be two regularly branched three-sheeted covering Riemann surfaces defined by two equations $y^3=G_2(z)$ and $u^3=g_2(w)$, respectively, where G_2 and g_2 are two entire functions having no zero other than an infinite number of simple or double zeros.

Next (§6∼§7) we shall discuss on the existence of analytic mappings between ultrahyperelliptic surface and our Riemann surface. Then we can conclude that there is no analytic mapping.

Finally (§8∼§10) we shall treat the case where S is a closed and regularly branched three-sheeted (or two-sheeted) covering Riemann surface.

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§2. Let $\mathfrak{p}_{R_3}$ and $\mathfrak{p}_{S_3}$ be the projection maps $(z, y)\to z$, $(w, u)\to w$, respectively. Let φ be an analytic mapping from R_3 into S_3.

DEFINITION. If φ satisfies $\mathfrak{p}_{S_3}\circ\varphi(p_1)=\mathfrak{p}_{S_3}\circ\varphi(p_2)=\mathfrak{p}_{S_3}\circ\varphi(p_3)$ for every distinct p_i $(i=1, 2, 3)$ with $\mathfrak{p}_{R_3}(p_1)=\mathfrak{p}_{R_3}(p_2)=\mathfrak{p}_{R_3}(p_3)$, then we say that φ satisfies the *rigidity of projection map* or φ is a *rigid* analytic mapping from R_3 into S_3. Similar definition for the rigidity of projection map may be given for an arbitrary analytic function on R_3 (or S_3). If the condition is not satisfied, then we say that φ is a non-rigid analytic mapping or function.

Here we shall prove the following theorem:

THEOREM 1. *There is no non-rigid analytic mapping from R_3 into S_3.*

In order to prove this theorem we need the following lemma on algebroid functions.

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