ON THE EQUIVALENCE OF LOCAL HOLOMORPHY AND LOCAL HOLOMORPHIC CONVEXITY IN TWO-DIMENSIONAL NORMAL COMPLEX SPACES

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A subdomain (or an open subset) X' of a complex space X is called a *subdomain* (an *open subset*) *of holomorphy of* X *of a holomorphic function* f *in* X', or shortly a *subdomain* (an *open subset*) *of holomorphy of* X, if f can not be analytically continued in any boundary point of X' relative to X. A subdomain X' of a complex space X is called a *subdomain of local holomorphy* (a *locally holomorphically convex subdomain*) if there exists a neighbourhood U for any boundary point of X' relative to X such that $U\cap X'$ is an open subset of holomorphy (a holomorphically convex open subset) of X. In the present paper we shall prove that a subdomain X' of a two-dimensional normal complex space X is a subdomain of local holomorphy of X if and only if it is a locally holomorphically convex subdomain of X.

§1. Application of Weyl's theorem.

Let K be a subset of a complex space Y. We shall denote by $\tilde{K}$ the set of all $x\in Y$ satisfying the following condition:

$$|f(x)|\leqq \sup_{y\in K}|f(y)|$$

for all holomorphic functions f in Y.

$\tilde{K}$ is called the *envelope of holomorphy of* K with respect to Y. If the envelope of holomorphy of any compact subset of a complex space Y is compact, Y is called *holomorphically convex*.

LEMMA 1. *Let K be a subset of a complex space Y with $\tilde{K}\subsetneqq Y$ and $\Delta=\{x_1, x_2, \cdots, x_s\}$ be a finite subset of $Y-\tilde{K}$. Then there exists a holomorphic function f in Y satisfying the following condition*:

$$\sup_{y\in K}|f(y)|<1<\inf_{y\in\Delta}|f(y)|.$$

Proof. For any $j=1, 2, \cdots, s$, there exists a holomorphic function f_j in Y satisfying the following condition:

$$\sup_{y\in K}|f_j(y)|<1/s<2<|f_j(x_j)|$$

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