A REMARK ON THE SPACE OF CLOSED RIEMANN SURFACES WITH ORDINARY WEIERSTRASS POINTS

By Yasushi Miyahara

It is known that the space of hyperelliptic Riemann surfaces of genus $g(\geqq 2)$ forms a $(2g-1)$-dimensional complex-analytic submanifold of the Teichmüller space T_g (cf. [2], [9]). In the paper [10], Rauch generalized this result as follows: the space of closed Riemann surfaces of genus $g(\geqq 2)$ having Weierstrass points where the first non-gap values is $n(\leqq g)$ forms an $(n+2g-3)$-dimensional complex-analytic submanifold of T_g. For the proof he used the Garabedian deformation.

We want to discuss the same problem by making use of deformations by Beltrami differentials introduced by Bers [6] (see also [3]). We have succeeded for the case $n=g$.

We remark that a related problem has been discussed by Bers [7] by using quasi-Fuchsian groups.

1. We begin with the statement of our result. In the present paper we consider only closed Riemann surfaces of a given genus $g(\geqq 2)$. Let S_0 be such a surface fixed once for all. We denote by σ a homotopy class of sense-preserving homeomorphisms of S_0 onto another S, and call the pair (S, σ) a marked Riemann surface. Two marked Riemann surfaces (S, σ) and (S', σ') are said to be conformally equivalent if the homotopy class $\sigma'\sigma^{-1}$ contains a conformal mapping of S onto S'. We denote by $<S, \sigma>$ the conformal equivalence class of (S, σ), and call the set of all $<S, \sigma>$ the Teichmüller space, which will be denoted by T_g. For given $<S_1, \sigma_1>$ and $<S_2, \sigma_2>$, there exists only one quasiconformal mapping f of S_1 onto S_2 which minimizes the maximal dilatation in the homotopy class $\sigma_2\sigma_1^{-1}$ (cf. [1], [5], [11]). We define the distance between two elements by

$$d(<S_1, \sigma_1>, <S_2, \sigma_2>)=\log K(f),$$

where $K(f)$ is the maximal dilatation of the mapping f. A topology on T_g is induced by this metric.

Let n be a positive integer and P_0 be a point of S. If no meromorphic function exists on S having as its only singularity a pole of order n at P_0, we say that n is a gap value at P_0, or that the point P_0 has a gap value n. It is known that there exist exactly g gap values at each point P_0. A point P_0 is called a Weierstrass point of S if it has a gap value n greater than g. There are only a finite number of Weierstrass points on S. Consequently, except a finite number of points on S,

Received February 11, 1965.