ON ULTRAHYPERELLIPTIC SURFACES

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§ **1.** Let R be an open Riemann surfaces. Let $\mathfrak{M}(R)$ be a family of non-constant meromorphic functions on R. Let f be a member of $\mathfrak{M}(R)$. Let $P(f)$ be the number of Picard's exceptional values of f, where we say α a Picard's value of f when α is not taken by f on R. Let $P(R)$ be a quantity defined by

$$\sup_{f\in\mathfrak{M}(R)} P(f).$$

In general $P(R)\geqq 2$. In [4] we showed that this was an important quantity belonging to R for a criterion of non-existence of analytic mapping.

Now let R be an ultrahyperelliptic surface, which is a proper existence domain of a two-valued algebroid function $\sqrt{g(z)}$ with an entire function $g(z)$ of z whose zeros are all simple and are infinite in number. Then by Selberg's generalization of Nevanlinna's theory we have $P(R)\leqq 4$. Further we showed that $P(R)$ was equal to 2 in almost all cases of ultrahyperelliptic surfaces, that is, we had the following result: If $g(z)$ is of non-integral finite order, then $P(R)=2$. In the present paper we shall establish the existence of an ultrahyperelliptic surface R with $P(R)=3$. The existence of the surfaces with $P(R)=4$ is evident, however we need a characterization of these surfaces with $P(R)=4$ for our purpose. We do not give any characterization of the ultrahyperelliptic surfaces with $P(R)=3$.

§ **2. A lemma on the number of simple zeros of the function** $e^{h(z)}-\nu$. In the sequel we need a property of the function $e^{h}-\nu$ on the number of simple zeros several times. Let T, m, N, N_1, $\bar{N}$ and S be the quantities defined in Nevanlinna's theory [3]. Let $N_2(r;\ a, f)$ and $\bar{N}_1(r;\ a, f)$ be the N-functions with respect to the simple a-points and to the multiple a-points of the indicated function f, which is counted only once, respectively.

Lemma. *Let h be an arbitrary given entire function of z. Then we have*

$$\varlimsup_{r\to\infty}\frac{N_2(r;\ \nu, e^h)}{T(r;\ e^h)}=1$$

for every non-zero constant ν.

Proof. By Nevanlinna's second fundamental theorem we have

$$T(r, e^h)<N(r;\ 0, e^h)+N(r;\ \infty, e^h)+N(r;\ \nu, e^h)-N_1(r;\ e^h)+S(r),$$

$$S(r)<O(\log rT(r, e^h))$$

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