NOTE ON A COUSIN-II DOMAIN OVER C^2

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Dedicated to Professor A. Kobori on his sixtieth birthday

Serre [7] gave a canonical exact sequence

$$0 \to Z \to \mathfrak{O} \to \mathfrak{O}^* \to 0$$

where Z is the additive group of all integers and $\mathfrak{O}$ and $\mathfrak{O}^*$ are, respectively, the sheaves of all germs of holomorphic mappings in a complex plane C and $\mathrm{GL}(1, C)$. Therefore we have an exact sequence of cohomology groups

$$H^1(X, Z) \to H^1(X, \mathfrak{O}) \to H^1(X, \mathfrak{O}^*) \to H^2(X, Z) \to H^2(X, \mathfrak{O}).$$

Hence $H^1(X, \mathfrak{O}^*) = H^1(X, Z) = 0$ and $H^1(X, \mathfrak{O}) = H^2(X, Z) = 0$ imply, respectively, $H^1(X, \mathfrak{O}) = 0$ and $H^1(X, \mathfrak{O}^*) = 0$. Taking Cartan [3]-Behnke-Stein [1]'s theorem into account, we see that any domain (D, φ) over C^2 with $H^1(D, \mathfrak{O}^*) = H^1(D, Z) = 0$ is a domain of holomorphy over C^2. Therefore, as we remarked in the previous paper [4], Thullen [9]'s example $E = C^2 - \{(0, 0)\}$ is a Cousin–II domain with $H^1(E, \mathfrak{O}^*) \neq 0$. In the present paper we shall prove that any domain (D, φ) over C^2 satisfies $H^1(D, \mathfrak{O}^*) = 0$ if and only if (D, φ) is a domain of holomorphy over C^2 with $H^2(D, Z) = 0$. Therefore any Cousin–II domain (D, φ) over C^2 which is not a domain of holomorphy over C^2 is always an example of a Cousin–II domain with $H^1(D, \mathfrak{O}^*) \neq 0$.

Let φ be a holomorphic mapping of a complex manifold D in C^n such that φ is locally a biholomorphic mapping. Then (D, φ) is called a *domain over* C^n. Let (D_1, φ_1) and (D_2, φ_2) be domains over C^n. If there exists a holomorphic mapping λ of D_1 in D_2 such that $\varphi_1 = \varphi_2 \circ \lambda$, (D_1, φ_1) is called a *domain over* (D_2, φ_2). Moreover, if there exists a neighbourhood U of x for any $x \in D_2$, such that λ is a biholomorphic mapping of each connected component of $\lambda^{-1}(U)$ onto U, then (D_1, φ_1) is called a *covering manifold of* (D_2, φ_2). For any domain (D, φ) over C^n, we can uniquely construct a covering manifold $(D^\sharp, \varphi^\sharp)$ of (D, φ) such that the fundamental group $\pi_1(D^\sharp)$ of $D^\sharp$ vanishes. This $(D^\sharp, \varphi^\sharp)$ is called a *universal covering manifold of* (D, φ). If (D, φ) coincides with its universal covering manifold, (D, φ) is called *simply connected.*

LEMMA 1. *Let (D, φ) be a domain over C^n and (D', φ') be its covering manifold. Then (D, φ) is a domain of holomorphy over C^n if and only if (D', φ') is a domain of holomorphy over C^n.*

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