PICARD'S THEOREM ON SOME RIEMANN SURFACES

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Dedicated to Professor K. Kunugi on his sixtieth birthday

1. Introduction.

In the present paper we shall establish the Picard theorem on some Riemann surfaces with automorphisms. Here we shall adopt a special method based on the Schottky theorem and the most far-reaching method due to Nevanlinna-Selberg. We shall roughly say that a class of meromorphic functions is exceptional if its any member has unreasonably many exceptional values. This nomenclature has no meaning in some cases when we impose the conditions guaranteeing the presence of an essential singularity or some growth conditions. The most important and well-known example of the exceptional class is that of functions of bounded type in $|z|<1$. Anyhow it is important to determine and to study the exceptional class in the various cases.

In order to investigate and to determine the number of Picard's exceptional values and the exceptional class of functions it is necessary to prove the existence of the fundamental functions in some cases. The functions play an essential role in the respective cases.

We shall make free use of the notations in [4], [6] and [7]. Any quantities in [7] and in [4] are distinguished from those in [6] by the subscripts A and P, respectively. In a way we shall give some remarks on the general value distribution theory, especially on the general defect relation.

Let W be a Riemann surface admitting a conformal transformation group G_n onto itself, which is a free abelian group with n generators $T_1, \cdots, T_n$. Further we assume that W has only one ideal boundary point defined by $\gamma=\lim_{m\to\pm\infty} T_j{}^m p, j=1, \cdots, n$, when $n\geqq 2$ and just two defined by $\gamma_1=\lim_{m\to+\infty} T^m p$ and $\gamma_2=\lim_{m\to+\infty} T^{-m} p$ when $n=1$ and that W is an unramified abelian covering surface of a closed Riemann surface. This class of surfaces is denoted by $\mathfrak{G}_n$. Then $W\in O_G$ if $W\in\mathfrak{G}_n$, $n=1, 2$, and $W\notin O_G$ but $W\in O_{AD}$ if $W\in\mathfrak{G}_n$, $n\geqq 3$ [3].

We shall adopt an exhaustion $\{W_a\}$ of W whose member W_a is the interior of a set defined by

$$\sum_{\sum_{j=1}^{n}|m_j|\leqq a} \prod_{j=1}^{n} T_j{}^{m_j} R^*,$$

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