A REMARK ON THE GENERALIZATION OF HARNACK'S FIRST THEOREM

By Yoshikazu Hirasawa

1. In the previous papers [1], [2], we gave some uniqueness conditions for the solution of the Dirichlet problem concerning semi-linear elliptic equations of the second order

$$L(u)\equiv\sum_{i,j=1}^{m}a_{ij}(x)\frac{\partial^2 u}{\partial x_i\partial x_j}=f(x, u, \nabla u), \tag{1. 1}$$

and under one of those uniqueness conditions, Harnack's first theorem was extended to the solution of the equation (1. 1). It was the case where the function $f(x, u, p)$ was non-decreasing with respect to u. In the present paper, we consider the case where the function $f(x, u, p)$ has not necessarily the above-mentioned property, and since Harnack's first theorem for solutions of the elliptic differential equation is really based on the *continuous dependence* of solutions upon the boundary data, we will here treat of this dependence.

Regarding the notations used in the present paper, confer the above-cited papers.

2. Let D be a bounded domain in the m-dimensional Euclidean space and let the differential operator $L(u)$ be of elliptic type in the domain D. In the present paper, we always suppose that the function $f(x, u, p)$ is defined in the domain

$$\mathfrak{D}=\{(x, u, p);\ x\in D,\ |u|<+\infty,\ |p|<+\infty\}.$$

For the sake of comparison with the later discussion, we first mention:

THEOREM 1. *Let the function $f(x, u, p)$ fulfill the following condition: For $\bar{u}>u$ and any p, q, we have*

$$f(x, \bar{u}, q)-f(x, u, p)\geqq-\alpha_0(x)(\bar{u}-u)-\alpha_1(x)\,|q-p|, \tag{2. 1}$$

where $\alpha_0(x)$ and $\alpha_1(x)$ are functions defined in D. And suppose further that there exists a function $\omega(x)$ belonging to $C^2[D]\cap C[\bar{D}]$, which is positive in $\bar{D}$ and satisfies the inequality

$$\alpha_0(x)\omega(x)+\alpha_1(x)\,|\nabla\omega(x)|\,+L(\omega(x))<0. \tag{2. 2}$$

Received January 24, 1963.