ON CURVATURES OF SPACES WITH NORMAL GENERAL CONNECTIONS, I

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In a previous paper [7], the curvature tensor of a space with a general connection was defined by formulas analogous to the classical ones for the spaces with affine connections. As is well known, the Ricci's formula

$$V^j{}_{,hk} - V^j{}_{,kh} = -R_i{}^j{}_{hk} V^i$$

is fundamental for the theory of differential geometry in the large, for instance, the holonomy group.

In this paper, the author will investigate the formula for spaces with normal general connections, making use of the results obtained in [12] regarding basic curves in such spaces. He will use the notations in [8], [10], [11] and [12].

§1. The curvature tensor of a space with a general connection.

Let $\mathfrak{X}$ be an n-dimensional differentiable manifold with a general connection Γ written in terms of local coordinates u^i as

$$\Gamma = \partial u_j \otimes (P^j_i d^2 u^i + \Gamma^j_{ih} du^i \otimes du^h).$$

By (6.28) in [7], the components of the curvature tensor of the space are given by

$$\begin{aligned} R_i{}^j{}_{hk} = &\left\{ P^j_l \left(\frac{\partial \Gamma^l_{mk}}{\partial u^h} - \frac{\partial \Gamma^l_{mh}}{\partial u^k} \right) + \Gamma^j_{lh} \Gamma^l_{mk} - \Gamma^j_{lk} \Gamma^l_{mh} \right\} P^m_i \\ &- \delta^j_{m,h} \Lambda^m_{ik} + \delta^j_{m,k} \Lambda^m_{ih}, \end{aligned} \tag{1.1}$$

where

$$\Lambda^j_{ih} = \Gamma^j_{ih} - \frac{\partial P^j_i}{\partial u^h}$$

and δ^j_i are the Kronecker's δ. The formulas can be written as follows:

$$\begin{aligned} R_i{}^j{}_{hk} = &\left\{ P^j_l \left(\frac{\partial \Lambda^l_{mk}}{\partial u^h} - \frac{\partial \Lambda^l_{mh}}{\partial u^k} \right) + \Gamma^j_{lh} \Gamma^l_{mk} - \Gamma^j_{lk} \Gamma^l_{mh} \right\} P^m_i \\ &- (\Gamma^j_{lh} P^l_m - P^j_l \Lambda^l_{mh}) \Lambda^m_{ik} + (\Gamma^j_{lk} P^l_m - P^j_l \Lambda^l_{mk}) \Lambda^m_{ih} \end{aligned}$$

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