ON THE SYSTEM OF INTEGRAL EQUATIONS OF VOLTERRA TYPE WITH INFINITELY MANY UNKNOWN FUNCTIONS

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The purpose of this paper is to consider the continuous solutions of an enumerably infinite system of integral equations of Volterra type with singularity; that is, the equations

$$
\text{(A)} \qquad \begin{aligned} xU_j(x) &= \int_0^x \sum_k a_{jk}(x, t) U_k(t)\, dt + b_j(x), \quad \text{or equivalently,} \\ &= \int_0^x F_j(x, t, U_1(t), U_2(t), \cdots)\, dt + b_j(x) \quad (j = 1, 2, \cdots, \infty). \end{aligned}
$$

Here we shall define, whenever

$$xU(x) = \int_0^x F(x, t, U(t))\, dt,$$

that

$$U(0) = \lim_{x \to 0} \frac{1}{x} \int_0^x F(x, t, U(t))\, dt.$$

In this paper we shall discuss the existence of continuous solutions of the above system by an argument similar to that used by Pogorzelski in [2], and apply the result to differential and integral equations.

In the first place, we shall state the following Property-N of normal-determinant and the theorem on which we base our argument.

Normal-determinant (N-determinant) [3]. An infinite determinant

$$|(\mathrm{A})| = |(\delta_{jk} + a_{jk})| \quad (j, k = 1, 2, \cdots),$$

where δ_{jk} is the Kronecker symbol, is called an normal- or an N-determinant if $S = \sum_{j,k} |a_{jk}|$ converges. The fundamental theorem on the solution of an infinite system of linear equations reads as follows:

Property-N: In the infinite system of linear equations

$$\sum_k (\delta_{jk} + a_{jk}) x_k = b_j \quad (j = 1, 2, \cdots),$$

suppose that the determinant $|(\mathrm{A})|$ is normal and distinct from zero, and that $|b_j| < b$ $(0 < b < \infty;\ j = 1, 2, \cdots)$. Then among all bounded sequences of numbers $(x_1, x_2, \cdots)$ there exists one and only one solution given by

$$x_j = \sum_k b_k |(D_{kj})| / |(\mathrm{A})| \quad (j = 1, 2, \cdots),$$

where $|(D_{kj})|$ is the co-factor of $\delta_{kj} + a_{kj}$ in $|(\mathrm{A})|$ for every k.

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