ON SOME LOCAL PROPERTIES OF FIBRED SPACES

BY K. YANO AND E. T. DAVIES

One of the most fruitful ideas in differential geometry is the idea exploited by E. Cartan of attaching a space to every point of a certain base space B. The attached space in Cartan's work is usually a homogeneous space F such that every point of F is equivalent to any other point under the action of a certain (structure) group G which operates transitively in F. The notion of connection as developed by Cartan consists of the establishment of a correspondence between the spaces F attached to two infinitely near points and the connection is called euclidean, affine, or projective according as the group G is the orthogonal, affine or the projective group. This conception of Cartan's has led to the modern notion of a fibre bundle developed mainly by Ehresmann, Chern and Lichnerowicz to whose fundamental works we refer. The homogeneous space F_P attached to a certain point P of the base space B is called the fibre. The spaces F_P attached to points of the base space are all homeomorphic to a certain type fibre F. The so-called bundle space E to which this leads is a leaved manifold whose dimension is the sum of the dimensions of the base space and of the fibre. Compound manifolds of a very similar kind have also been extensively treated by Wagner [25] but his point of view is somewhat different. In fibre bundle theory the three spaces E, B and F are differentiable manifolds. The fibres homeomorphic to the type fibre F are holonomic subspaces of the bundle space E and in local coordinates can be expressed by finite equations satisfied by the local coordinates of E in that region. The tangent space to E at any point can then be decomposed into two complementary spaces, one of which is tangent to the fibre and the other is a non-holonomic subspace (or a non-integrable distribution in the terminology of Chevalley [4]) transversal to the fibre. It has now become customary to refer to a vector tangent to the fibre as a 'vertical' vector, and a vector belonging to the complementary transversal distribution as a 'horizontal' vector. The notion of connection is now often formulated in terms of these complementary subspaces of E.

In this paper the authors take the general space E to be a Riemannian space, or a space with a euclidean connection, or a space of paths. The fibres are differentiable subspaces of E which can be expressed locally in the form $f(\xi)=x$ where ξ are local coordinates in E. If a geometric object defined in E can be expressed locally in terms of x only that geometric object will be said to be *induced* in the base space.

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