ON EXTREMAL QUASICONFORMAL MAPPING

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Let $\{T_n\}$ be a sequence of quasiconformal mappings of a domain D onto a domain Δ converging uniformly to a quasiconformal mapping T of D onto Δ. Let K_n and K be the maximal dilatations of T_n and T, respectively, then K_n tends to K when n tends to infinity. Our first purpose in the present note is to establish that the quotient of two complex derivatives q_{T_n}/p_{T_n} of T_n converges to q_T/p_T of T in a certain sense under the above situation. Our result may possibly be not original. However, so far as we concern, there are no papers stating it explicity. Our second purpose is to establish that there exists an extremal quasiconformal mapping in a family with a boundary correspondence and it satisfies a differential equation of Beltrami type with some remarkable restrictions. To this end, we shall apply the result stated in the first part. According to the result obtained in this case, one can recognize that there exists an essential difference between the cases treated previously by Teichmüller [10, 11] and recently by Ahlfors [1] and a case presented here. Then there arise many unsolved problems, all of which are perhaps very difficult to settle. Situations are quite similar in a case with a countably infinite number of distinguished boundary points.

1. Definitions and known results on quasiconformal mappings.

Among various definitions of quasiconformality the one due to Pfluger-Ahlfors-Mori [1, 5, 6] is most convenient for our later purposes.

A topological mapping $w=T(z)$ of a planar region D onto another such region Δ is called quasiconformal with the parameter K, if (i) $w=T(z)$ preserves the orientation of the plane, and (ii) for any quadrilateral Ω contained in D together with its boundary, it satisfies

$$\operatorname{mod} T(\Omega) \leqq K \operatorname{mod} \Omega,$$

where K is a constant $\geqq 1$ and $\operatorname{mod}\square$ denotes the modulus of the indicated quadrilateral $\square$. The infimum of K satisfying the above condition is called the maximal dilatation K_T of the mapping T.

Mori proved in his theorem 1 [5, 6] that (i) $w=T(z)$ is totally differentiable almost everywhere in D; (ii) at each totally differentiable point z

$$(|p_T|+|q_T|)^2 \leqq K_T(|p_T|^2-|q_T|^2),$$

where $p_T=\partial T/\partial z$ and $q_T=\partial T/\partial \bar{z}$; (iii) $w=T(z)$ is absolutely continuous in Tonelli's sense, that is, for almost all $y=y_0$, the function $T(x, y_0)$ is absolutely

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