# ON GENERALIZATION OF FROSTMAN'S LEMMA AND ITS APPLICATIONS

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## 1. Introduction.

The generalized Riemann-Liouvill's integral has been given by M. Riesz. By the use of a lemma which is a consequence of Riemann-Liouville's integral, Frostman [1] has proved his fundamental theorem on the energy integral. In 1938, M. Riesz [4] has given the relations between the Riemann-Liouville's integrals and potentials.

In the present paper, we shall give a generalization of Frostman's lemma in a higher dimensional space and some analogous lemmas in two dimensional space. Also we shall give some examples which show us how to apply them. The author is much indebted to Professor Y. Komatu who gives him many useful advices.

## 2. On generalized Riemann-Liouville's integrals.

Let $r_{PQ}=r_{QP}$ be the distance between P and Q in the $m$-dimensional euclidean space $\Omega_m$ $(m \geqq 1)$, and $\alpha$, $\beta$ be positive numbers. Then we define the integral by M. Riesz:

$$I^{\alpha}f(\mathrm{P}) = \frac{1}{C_m(\alpha)}\int_{\Omega_m} f(\mathrm{Q})r_{\mathrm{PQ}}^{\alpha-m}d\mathrm{Q}, \tag{A}$$

where

$$C_m(\alpha) = \pi^{\frac{m}{2}}\frac{2^{\alpha}\Gamma\left(\frac{\alpha}{2}\right)}{\Gamma\left(\frac{m-\alpha}{2}\right)} \tag{B}$$

and $d\mathrm{Q}$ denotes the volume element. Here $f(\mathrm{Q})$ is continuous and satisfies the condition that the above integral (A) should converge absolutely.

For example, in order that (A) converges near the point $\mathrm{P}=\mathrm{Q}$, it is necessary that $\alpha>0$ while the convergence near the point at infinity depends on the behavior of $f(\mathrm{Q})$. If $f(\mathrm{Q})$ is a continuous function which behaves like $e^{-cr}$ at infinity, $I^{\alpha}f(\mathrm{Q})$ exists when $\alpha>0$ and it represents a continuous function of $\alpha$. If, however, $f(\mathrm{Q})$ is a continous function which behaves like $1/r^{\kappa}$ $(\kappa>0)$ at infinity, $I^{\alpha}f(\mathrm{Q})$ exists when $0<\alpha<\kappa$ and it represents a continuous function of $\alpha$ in the interval $0<\alpha<\kappa$.

Concerning Riesz's operator $I^{\alpha}$, the fundamental results are mentioned as follows:

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