ON CONTINUABILITY OF BILINEAR DIFFERENTIALS

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Schiffer and Spencer [3] have derived a condition under which bilinear differentials are continuable. In this paper, applying the results due to Aronszajn [1], we shall give a condition in terms of positive definite kernels.

Let D be a domain in the z-plane. A function $\psi(z, \bar{\zeta})$ of z, $\zeta \in D$ is called a Hermitian kernel on D, if it satisfies $\psi(z, \bar{\zeta}) = \overline{\psi(\zeta, \bar{z})}$. If for any points $y_1, y_2, \cdots, y_n \in D$ and any complex numbers $\xi_1, \xi_2, \cdots, \xi_n$ the inequality

$$\sum_{i,j=1}^{n} \psi(y_i, \bar{y}_j)\xi_i\bar{\xi}_j \geqq 0 \qquad (n=1, 2, \cdots)$$

is satisfied, then $\psi(z, \bar{\zeta})$ is called a positive definite kernel on D. Further, we denote by P_D the aggregate of all positive definite kernels $\psi(z, \bar{\zeta})$, which are analytic in z, $\bar{\zeta}$ respectively. Let ψ, $\varphi \in P_D$. We denote $\varphi \ll \psi$ if for any points $y_1, y_2, \cdots, y_n \in D$ and any complex numbers $\xi_1, \xi_2, \cdots, \xi_n$

$$\sum_{i,j=1}^{n} \psi(y_i, \bar{y}_j)\xi_i\xi_j - \sum_{i,j=1}^{n} \varphi(y_i, \bar{y}_j)\xi_i\xi_j \geqq 0 \qquad (n=1, 2, \cdots).$$

Now, generally, the following lemma is well known (cf. [4]).

LEMMA 1. *Let E be an abstract set. If a function $k(x, y)$ of $x, y \in E$ satisfies*

$$\sum_{i,j=1}^{n} k(y_i, y_j)\bar{\xi}_i\xi_j \geqq 0 \qquad (n=1, 2, \cdots)$$

for any points $y_1, y_2, \cdots, y_n \in E$ and any complex numbers $\xi_1, \xi_2, \cdots, \xi_n$, we can construct a Hilbert space which has $k(x, y)$ as its reproducing kernel.

Proof. Let F_1 be the family of functions f_1 which are of the form

$$f_1(x) = \sum_{j=1}^{n} \alpha_j k(x, y_j)$$

where $y_1, \cdots, y_n$ are any points of E, $a_1, \cdots, \alpha_n$ any complex numbers and n any natural number. Let the inner product be defined by

$$(f_1, g_1) = \sum_{j,i=1}^{\max(m,n)} \alpha_j\bar{\gamma}_i k(u_i, y_j), \qquad (f_1, f_1) = \|f_1\|^2,$$

where

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