ON FUCHS' RELATION FOR THE LINEAR DIFFERENTIAL EQUATION WITH ALGEBRAIC COEFFICIENTS

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1. In the theory of linear differential equations of Fuchsian type with rational coefficients, it is well known that the sum of the roots of indicial equations of an equation of this type is equal to

$$n(n-1)\left(\frac{m}{2}-1\right)$$

where n is the order of the equation and m is the number of its singular points. The purpose of this paper is to generalize this Fuchs' relation for the case when the coefficients of the equation are algebraic.

The equation we are to investigate is

$$\frac{d^n v}{dx^n}+p_1\frac{d^{n-1}v}{dx^{n-1}}+\cdots+p_n v=0 \tag{1}$$

where the coefficients $p_1, p_2, \cdots, p_n$ are all supposed to be one-valued and meromorphic functions on a Riemann surface $\mathfrak{F}$ of an algebraic function $y=\varphi(x)$ with genus p and consisting of r sheets.

We denote by

$$\mathfrak{q}_1=(\alpha_1, \beta_1), \quad \cdots, \quad \mathfrak{q}_s=(\alpha_s, \beta_s);$$
$$\mathfrak{r}_1=(\infty, \gamma_1), \quad \cdots, \quad \mathfrak{r}_r=(\infty, \gamma_r);$$
$$\mathfrak{p}_1=(a_1, b_1), \quad \cdots, \quad \mathfrak{p}_m=(a_m, b_m)$$

the branch points of $\varphi(x)$, the points at infinity on $\mathfrak{F}$, and the singular points of the equation (1) respectively, where the notation (α, β) stands for the point of $\mathfrak{F}$ such that $x=\alpha$, $y=\beta$. For simplicity's sake, we assume that

$$\begin{aligned} &\mathfrak{p}_j \neq \mathfrak{q}_k, && \text{for } j=1, \cdots, m \text{ and } k=1, \cdots, s,\\ &\mathfrak{p}_j \neq \mathfrak{r}_k, && \text{for } j=1, \cdots, m \text{ and } k=1, \cdots, r,\\ &\mathfrak{q}_j \neq \mathfrak{r}_k, && \text{for } j=1, \cdots, s \text{ and } k=1, \cdots, r. \end{aligned} \tag{2}$$

2. We suppose that the equation (1) is of Fuchsian type. To derive the generalized Fuchs' relation for this equation, we must investigate the behaviour of the coefficient p_1 on $\mathfrak{F}$.

First, p_1 must have a pole of order at most 1 at $\mathfrak{p}_1, \cdots, \mathfrak{p}_m$. Accordingly it is expanded, in the neighbourhood of each $\mathfrak{p}_j$, in the form

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