ON A COEFFICIENT PROBLEM FOR ANALYTIC FUNCTIONS TYPICALLY-REAL IN AN ANNULUS

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Considering a class of analytic functions regular and univalent in an annulus whose Laurent coefficients are all real, Nehari and Schwarz [3] have given an estimation of Laurent coefficients for any function of the class. As they have also remarked there, the result remains valid for a slightly wider class, i. e. the class consisting of functions regular and typically-real in an annuls. Komatu [2] has then ameliorated the bound of this estimation by modifying Szász' method previously used for a similar problem on Taylor series. The result obtained is precise for all coefficients, as shown by extremal function.

In the present paper, we first introduce an integral representation valid for any function of the class under consideration.

Though it is a simple consequence of a well-known Villat's formula, we state its proof fully for the sake of completeness. By making use of this representation, we give an alternative proof for the estimation of the coefficients due to Komatu. Finally, it is shown that our present method enables us to determine all possible functions which are extremal for our coefficient problem.

THEOREM 1. *Let*

$$f(z)=\sum_{n=-\infty}^{\infty}a_nz^n \tag{1}$$

be a single-valued analytic function which is regular and typically-real in an annulus $q<|z|<1$ *and satisfies* $\Im f(z)\cdot\Im z>0$ *for* $\Im z\neq 0$. *Then there holds an integral representation*

$$\begin{aligned}f(z)&=\int_0^\pi\frac{\zeta(i\lg z+\varphi)-\zeta(i\lg z-\varphi)}{2\sin\varphi}\,d\rho(\varphi)\\&\quad-\int_0^\pi\frac{\zeta_3(i\lg z-\varphi)-\zeta_3(i\lg z-\varphi)}{2\sin\varphi}\,d\tau(\varphi)+c\end{aligned} \tag{2}$$

where $\rho(\varphi)$ *and* $\tau(\varphi)$ *are real-valued functions satisfying the conditions*

$$\begin{aligned}&d\rho(\varphi)\geqq 0,\qquad \int_0^\pi d\rho(\varphi)=a_1-a_{-1};\\&d\tau(\varphi)\geqq 0,\qquad \int_0^\pi d\tau(\varphi)=a_1q-a_{-1}q^{-1}\end{aligned} \tag{3}$$

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